1. Introduction

The WDM has been introduced to increase the transmission capacity of existing optical links. Instead of using one several transmitter and receiver pairs are used over the same fibre at different wavelengths forming independent channels and overbridging the speed limitations of electronics. It has been soon recognised that the switching decision can be made according to the incoming wavelength without any processing of the data stream. In WDM based All-Optical Networks (AON, where the whole network including the user-to-network interface (UNI) is optical) a wavelength is assigned to a connection in such a way that each connection (wavelength) is handled (switched) in the optical domain without any electrical conversion during the transmission [1]. Wavelength (WL) reuse is allowed in parts of the network where that WL was not used. This AO WDM Network would require a huge number of different WLs. The technology sets the limit to around 40 different WLs per fibre in the 1550 nm window with 100 GHz (about 0.8 nm) spacing in the flat operating gain band of the present Erbium-Doped Fibre Amplifiers (EDFA: 1530-1560 nm) according to the recently completed ITU-T Recommendation G.692. For this reason WL conversions are needed. The most expensive way is to make optical WL shifting which ensures transparency of the network. Simpler and cheaper method is to do first opto-electrical conversion, electrical space-switching and then electro-optical conversion. This is the idea for implementing so called opaque networks, where systems using different sets of WLs are to be interconnected. The optimisation method proposed in this paper can be applied to both.

Many excellent papers deal with design, configuration and optimisation of WDM Networks. See, e.g., [2–8]. The widely accepted approach is to decompose the problem to the following sub-problems in given order. First, determine the virtual topology (route the light-paths); second, assign a wavelength to each light-path (WA); and third, route the traffic over the light-paths.

In [2] a heuristic (greedy) algorithm is proposed for WA followed by routing over established light-paths. In [3] is defined a bound for carried traffic in all-optical networks and shown that the proposed heuristic Routing and Wavelength Assignment (RWA) algorithm gives results very close to this bound. A performance study has been carried out. In [4] the mathematical formulation of the design problem is given along with heuristics for solving the sub-problems relaxing some of the constraints one-by-one. In [5] there is also given a heuristic algorithm, and is showed that networks with and without wavelength converters require about the same number of wavelengths. A hybrid solution is
also proposed where wavelengths are electrically regenerated in some specific nodes. In [6] multi-commodity flow model with randomised rounding is applied followed by graph colouring algorithms. In [7] an algorithm is given for Light-path routing by transforming the network to a special structure called wavelength graph. In this graph costs are assigned to edges and shortest path algorithms are run. The optimality of the algorithm is also proved. In [8] design principles of Optical Networks are explored. The design is formulated as an optimisation problem with two objectives to be solved by heuristics. The (weighted) average delay through the network is minimised, while the total carryable traffic over the network is maximised. In [9] the extended layered graph is used which is a bit similar to our model, but not flexible enough. In [10] the optical path routing strategies for WDM networks are investigated. The performance of networks with and without wavelength converters is evaluated. In [11] is described a method for planning WDM layer for carrying ATM traffic over it with the survivability constraint using Tabu Search. In [12] a reconfigurable OADM is demonstrated. In [13] hitless reconfiguration of WDM networks is investigated. [14] gives a performance evaluation and comparison of WDM networks with and without wavelength interchange capability. [15] proposes an algorithm for simultaneous Routing and Wavelength Assignment on a Path Graph. Channel capacities are not taken into account. In [16] an exact linear programming formulation is presented and the “closest” virtual topology is chosen for reconfiguration. [17] investigates the performance of partial reconfiguration on a SDM/WDM architecture. [18] proposes a model and algorithm for finding the globally optimal WL assignment with high probability using generally applicable heuristics for global optimisation.

Our subject is to configure the light-path system optimally without decomposing the problem into subproblems. This improves the quality of results, but on the other hand the complexity of the problem grows rapidly.

As the optimisation result we decrease the traffic to be processed and carried by electrical facilities over-bridging the speed limits of electronics. Since a considerable part of the load of electrical, e.g., ATM switches is undertaken by the optical switches much larger networks with higher loads can be realised by the current technology.

First, we present the model of the network with different node-types and then we formulate the problem as an Integer Linear Program (ILP).
2. Light-Link Graph - The Model of the Network

The task was to provide a general model for configuration of WDM networks with different types of nodes and arbitrary topologies. Although the most popular topology is ring or interconnected rings, the model must be able to handle any specific or mesh topology. The nodes can also be quite different: Optical Add-and-Drop Multiplexers (OADM), Optical Cross-Connects (OXC) with full or limited (optical or opto-electrical) WL conversion or even an Electrical Cross Connect (EXC). The protection strategies can also be quite different. All these aspects are taken into account in the proposed model. First the link model is described followed by models of different nodes. In this section we assume that all traffic demands are bidirectional and symmetrical. In this case the network can be modelled by an undirected graph. The model can be simply generalised for un-symmetrical demands, by using directed graphs. In later case the model is more complex and for this reason the algorithms will run slower.

2.1. Model of Links

![Diagram](ondm2.tex)

*Figure 1. Modelling edges.*

A network consists of nodes, and links connecting the nodes. This can be modelled by a graph: a node is a vertex and a link is an edge. Having multiple WLs we will represent a WL of a link as an edge in the graph of wavelengths according to Figure 1 for the network proposed in [19]. To prioritise filling up WLs one-by-one we can assign slightly different weights to different channels of one link. For example, edges
representing WL1, WL2 and WL3 will have weights 101, 102 and 103 respectively.

2.2. Model of Nodes

A node is modelled by a subgraph. The subgraph-nodes are the switchports, while the weighted edges represent the costs of transitions, terminations, conversions, etc. There are different types of nodes. Models of nodes differ for these. Here will be shown some examples. In similar manner a model can be derived for any additional node-type. The models proposed here are similar to those described in [20], but those were used for setting up connections one-by-one using shortest path algorithms, while here is the emphasis on global simultaneous configuration requiring special node-models.

2.2.1. Optical Add-and-Drop Multiplexers: OADM

![Figure 2. Model of OADM Nodes.](image)

The OADM Nodes have in general two bi-directional ports (4 fibres). Their function is to transmit a WL channel or to terminate it and usually they do not allow WL-conversion.

The weights assigned to edges representing termination (e.g., 50) are higher than weights of transition (e.g., 25), because transition is preferred to termination. According to the proposed model (Figure 2) the traffic streams can enter or exit the OADM crossing vertex E or can be even re-multiplexed.

2.2.2. Electro-Optical Cross-Connect: EXC

In the model shown in Figure 3 each pair of nodes should be connected by an edge. All edges should have equal weights. Instead of using \( nxn \) edges we use \( n \) edges and one node. This simplifies the model. Each
incoming channel is converted to electrical domain switched by a space-switch and again converted to the optical domain. Each termination, transition or WL change of a light-path has the same cost (e.g., 25). Therefore all edges have the same weight (e.g., 25/2).

2.2.3. Optical Cross-Connect: OXC

An optical Cross-Connect has more than two ports, e.g., four bi-directional ports according to Figure 1. In an OXC a light-path can make transition to any output port which supports that WL, and that WL is not yet used. This OXC type (without WL change capability) will be referred to as *simple* OXC (see Figure 4). In this case one incoming channel can exit at any of the remaining output ports where that WL is supported and not yet used.

In some OXC devices WL translation (change) is also supported. This node will be called OXC. Its model is showed in Figure 5. For this node any incoming channel can exit at one of 11 remaining channels.
Now there are 3 possibilities for light-path transition since there are channels of the same wavelength on all ports in our example. For WL change there are 8 possibilities. It has higher cost (e.g., $2 \times 50 = 100$) comparing to the WL transition. WL change is modelled by conducting the traffic stream through node E.

![Diagram of an Optical Cross-Connect Node (with WL conversion)]

*Figure 5. Model of an Optical Cross-Connect Node (with WL conversion)*

In some cases the traffic stream termination is also among the functions of an OXC. In that case the model does not need any change. The only difference will be that there will be some traffic offered to that OXC node which can be modelled by offering traffic to node E and considering it as an end-node. In this case traffic-stream re-multiplexing capability is also required.

2.2.4. *Modelling opto-electro-optical conversions, multiplexing and re-multiplexing*

If we want to differentiate the simple wavelength-change from the electrical signal re-multiplexing a more complex model is needed. An example has been shown in Figure 6 for an OADM node for simplicity reasons, which can be extended to any other node-type. As can be seen node E has been substituted by a fully connected graph. In this case assigning costs to internal edges the costs of wavelength-change and signal re-multiplexing can be differentiated.

All-Optical WL conversion is not supported by all OXC. Therefore the optical signal is terminated and passed to the electrical layer where space switching or space switching with time switching (re-
Figure 6. Modelling opto-electro-optical conversions and re-multiplexing: complex model

multiplexing) is done and then the resulting electrical signal passed back to the optical layer.

In cross connects three levels of cross-connecting and switching are to be differentiated:

- WL transition - This is done by the optical layer without any processing. This is the preferred and cheapest function.

- WL translation - It can be done by the optical WL shifters, or by opto-electrical conversion, space-switching and electro-optical conversion. It is more expensive than the previous function, but still cheaper than the next one. Here is the switching very simple and no traffic stream processing is needed.

- multiplexing and re-multiplexing - In a larger WDM transport network there are considerably less available WLs per fibre than it would be needed for full interconnection of the end nodes by single-hop lightpaths. For this reason some of the traffic streams have to be multiplexed along a lightpath, i.e., in some cases the lightpath termination is not a traffic stream termination. In these cases time-division re-multiplexing is needed.

epsfig

3. Problem Formulation

It is algorithmically very complex to obtain globally optimal solution for the global simultaneous routing and wavelength assignment problem. (The problem can be expressed as well as configuration of the lightpath system of a WDM network.) This problem very likely belongs to the class of NP-hard problems [21], because its sub-problem, the Static Lightpath Establishment (SLE) has been shown to be NP-hard
Therefore it is righteous to use approximations. The proposed model enables solving the Routing and Wavelength Assignment problems simultaneously either using approximations or exact formulation.

The task was to find a shortest path in the obtained graph (which is built up of the link and node models) between all pairs of nodes simultaneously. There are alternatives for choosing the objective of the optimisation, e.g.:
1. Minimise the total number of used WLs per fibre.
2. Decrease the total amount of used resources at the optical layer.
3. Decrease the total amount of used resources and processing at the electrical layer.
4. Decrease the total amount of lost traffic. It is also possible to introduce a scale-up factor for all traffic demands. For example scaling up all traffic demands by 10% the network should still work properly.
5. Minimise the number of WL conversions in total and for each path.

Our objective function will optimise 3. and 5. simultaneously, as discussed in the next section.

4. ILP formulation

The above described problem can be formulated as an Integer Linear Program using the proposed models. For this purpose the undirected graph model will be used, which is less complex (i.e., needs less variables) than the model with directed graphs. This formulation has slight similarities with Minimal Cost Multicommodity Flow (MCMCF) problem formulation [23].

Objective:

$$\text{minimise } \sum_{w \in W} \sum_{o \in O} c_w b^o x^o_w$$

Subject to constraints:

$$\sum_{o \in O} x^o_w b^o \leq B_w \quad \forall w \in W$$  \hspace{1cm} (1)

$$\sum_{j \in A_i} x^o_{ij} = 1 \quad \forall i \in V_E, \forall o \in O$$  \hspace{1cm} (2)

$$\frac{1}{2} \sum_{j \in A_i} x^o_{ij} = z^o_{2i} \quad \forall i \in V \setminus V_E, \forall o \in O$$  \hspace{1cm} (3)

$$x^o_w \leq y_w \quad \forall w \in W, \forall o \in O$$  \hspace{1cm} (4)

$$y_w \leq \sum_{o \in O} x^o_w \quad \forall w \in W$$  \hspace{1cm} (5)
\[
\frac{1}{2} \sum_{j \in A_i} y_{ij} = z_{ij} \quad \forall i \in V \setminus V_E \quad (6)
\]
\[
x_w^o \in \{0, 1\} \quad \forall w \in W, \quad \forall o \in O \quad (7)
\]
\[
y_w \in \{0, 1\} \quad \forall w \in W \quad (8)
\]
\[
z_{ii}^o \in \{0, 1\} \quad \forall i \in V \setminus V_E, \quad \forall o \in O \quad (9)
\]
\[
z_{yi} \in \{0, 1\} \quad \forall i \in V \setminus V_E \quad (10)
\]

\(o \in O\) OD pair demands (commodities) \(o\) of the set of all commodities \(O\)

\(w \in W\) wavelength-links \(w\) of the set of all wavelength-links \(W\)

\(x_w^o, x_{i,j}^o\) flow indicator of commodity \(o\) over wavelength-link \(w\) or link \((i - j)\)

\(y_w\) light-path indicator

\(z\) indicator for emulating the condition “equal to 0 or 2”

\(\beta^o\) bandwidth requirement for traffic stream \(o\)

\(c_w\) cost for using wavelength-link \(w\) or link \((i - j)\)

\(B_w\) bandwidth (capacity) of wavelength-link \(w\) or link \((i - j)\)

\(i, j\) node-indices in the model

\(A_i\) set of nodes adjacent to node \(i\)

\(V\) set of all nodes (vertices) in the model

\(V_E \subset V\) set of all end-nodes (vertices)

The objective is to minimise the number of hops for each traffic demand weighted by the required capacity of that traffic stream and by the cost of using those light-links subject to the following constraints. The first constraint states that the amount of traffic using a light-link may not exceed the capacity of that light-link. The second constraint ensures that traffic-streams are to be terminated at end-nodes. Third the traffic flows must be conserved at each non-end node. Constraints 4 and 5 guarantee that traffic streams may use only available light-paths, and a light-path will be established only if it is needed for carrying a traffic flow. Sixth constraint expresses that a light-path can not branch. The last four constraints mean that all variables can take values 0 or 1.

The optimisation will result in a single-hop configuration (Wavelength-paths) whenever possible or in a multihop configuration with as few WL translations and re-multiplexing as possible, i.e., the largest possible part of the load of the electrical layer will be overtaken by the optical layer.

If the aim was to decrease the number of used WLs in total the objective can be expressed as

\[
\text{minimise } \sum_{w \in W} \left( c_w \sum_{o \in O} \beta^o x_w^o + y_w \right)
\]
as well.

5. Comments

In paper [5] the authors present different ways of formulating the problem for both single-hop and multihop lightpaths referred to as Wavelength-Path (WP) and Virtual Wavelength-Path (VWP) respectively. Among other methods they also use ILP. The drawback of the VWP formulation using ILP (and also of other methods applied to VWP) is that they implicitly assume pure electrical nodes, where not only electrical space-switching but also time-switching, i.e., re-multiplexing has to be performed. This approach degrades the Wavelength Routing network to a network employing WDM links and therefore it requires electrical (e.g., ATM) switches of large capacities increasing the costs and deteriorating the performance. The advantage of the method proposed in this paper is that it can differentiate various nodes with flexible functionality.

In paper [7] also a graph-model has been used but vertices are arranged in a matrix like grid, and edges representing lightpaths afterwards. Although the method is advantageous because of the applicability of fast shortest path algorithms, it does not take into account different node-types except the electrical switches. Furthermore, the method is used for routing of lightpaths only, not for configuration, i.e., simultaneous routing of all traffic demands.

The bin-packing problem, where we want to pack objects of different size into bins (of equal size) in optimal way is NP-hard. If the wavelength-channel capacities over all links are considered to be “bins” and we want to “pack” them optimally by traffic streams of different demands (different “objects”), we would have the same problem. However, our problem is even more complex, since there is “interaction” between “bins”, because loading one bin will induce loading one of the neighbouring bins, and if two neighbouring bins are loaded by the object of same type no other bins from the neighbourhood can be loaded by objects of that particular type. For the above reason this method is also NP-hard.

Solving the problem by any available ILP solver (e.g., LP_SOLVE or CPLEX) will be possible for very small networks only. The reason is that the number of both variables and constraints will grow by increasing the size of the network and this will result in exponential growth of the alternatives to be investigated by ILP software. For this reason ILP formulation does not solves the problem, but enables running efficient randomised “0-1” programming methods, e.g., Simulated Annealing,
Genetic Algorithm or Tabu Search - all using binary encoding. Based on the above formulation even more sophisticated heuristic methods can be used which exclude a part of the state-space which is not of interest, instead of using penalty term as a means of obeying constraints. Detailed description, numerical evaluation and comparison will be subject of another paper.

6. Conclusion

The novelties of this proposal are the general model for WDM networks which enables the IP formulation of the static RWA problem, and the IP formulation itself. The model is very flexible in sense of supporting various types of nodes and networks.

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Integer Linear Programming (ILP) formulation of the Optimal Lightpath-System Configuration Problem in Wavelength-Routing WDM Networks for arbitrary Node-Types and Topologies

Tibor CINKLER (cinkler@ttt-atm.ttt.bme.hu)
High-Speed Networks Laboratory
Department of Telecommunications and Telematics
Technical University of Budapest
Pázmány Péter sétány 1/D, H-1111 Budapest, Hungary
tel: +(36) 1 463-1861
fax: +(36) 1 465-3197

Abstract. An efficient and general graph-theoretic model is proposed which enables formulating the static Routing and Wavelength Assignment (RWA) problem as an Integer Linear Program (ILP). The topology of the physical layer and the type of each node (e.g., OADM, OXC or EXC) is assumed given with the traffic demands of each node-pair. The output of the optimisation is a light-path (a path with a wavelength assigned to it) for each traffic demand or a sequence of light-paths. The obtained result is optimal in sense of reduced resource usage on upper (electrical) layers, subject to constrained amount of capacity of each wavelength and limited number of wavelengths. The advantage of the proposed formulation is that it solves the Routing and Wavelength Assignment problems simultaneously.
