A Compact Mathematical Formulation for Shared Path Protection with General Shared Risk Groups*

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Abstract – This paper provides a compact mathematical formulation for shared protection in communication networks with bandwidth guaranteed tunnels. The formulation is characterized by taking the most general definition of the Shared Risk Group (SRG) with the complete routing information in the single failure scenario, which can facilitate solving the Shared Path Protection Problem on various network topologies and various graph models of networks. The general SRGs are essential for survivable routing in multi-layer grooming networks as well as in multi-domain networks. With the formulation, a general approach and a unified expression for solving the dynamic survivable routing problem can be developed. A case study is given at the end to verify the proposed formulation on a well studied special case, namely protecting against a single node or a single link failure.

Keywords – shared protection, least-cost survivable routing, Shared Risk Group (SRG).

I INTRODUCTION

Shared protection has been well recognized as a means providing the best compromise between the survivability and capacity-efficiency for the future high-speed communication networks [1-3]. With shared protection, working and the corresponding protection paths are Shared Risk Group (SRG)-disjointedly routed, in which the spare capacity along the protection paths is not configured during the normal operation. Once a failure hits any SRG that interrupts the working paths involved in the SRG, the network devices along the corresponding protection paths will be configured immediately using prioritized signalling such that the services can be restored within the least amount of time. In such a circumstance, different working paths can take the same spare capacity if they are not involved in any common SRG.

One of the most critical tasks in the protection and restoration mechanisms described above is to find an SRG-disjoint working and protection path-pair for a connection request which has been proved to be NP-complete. However many studies have focused on using heuristics and user-defined cost functions to derive working and protection paths respectively [1-2]. Moreover, an Integer Linear Program (ILP) can be formulated to find the least-cost working and protection path-pair provided with sufficient knowledge of global link-state [3,6]. In order to apply the above results in the future multi-layer and multi-service networks, a concise and general mathematical formulation that can consider different protection scenarios with more flexible definition of SRG is important and has little been addressed before. The study by Yu Liu [1,2] provides a matrix expression for spare capacity allocation, in which 100% restorability can be achieved for single domain single layer networks if each link in the network topology serves as an SRG. However, the expression fails if there is no one-to-one mapping between the arcs of the graph representation of the network and the SRGs, which is the case in multi-layer [4] or multi-domain [5] networks, or even in single layer single domain networks, if multiple network elements can be contained in a single SRG. Since in multi-layer networks the upper layer virtual topologies are embedded in the lower layer topology it is not straightforward what upper layer network elements belong to the same SRG. General SRG concept simplifies this dependence by simply grouping upper layer elements into SRGs without having to deal with both layers while routing the protection restoration paths. In case of multi-domain networks to achieve scalable routing the information of certain domains is aggregated and a simplified, aggregated view (a limited amount of information) is disseminated only through the network. The same holds for multi-provider networks, where the certain domains are operated by different operators. In this case network operators do not want to disseminate their confidential information on routing hiding also the information necessary for shared protection. Disseminating general SRG information can allow using shared instead of dedicated protection in these multi-domain multi-provider networks without violating the confidentiality requirements.

This paper generalizes the expression developed in [1,2], and aims to solve the SRG-disjoint routing problem using a compact mathematical formulation that considers the most general definition of SRGs. With the proposed mathematical formulation, the dynamic survivable routing for shared protection can be developed through a unified expression of matrix operations.

The paper is organized as follows. Section II provides the background knowledge for shared protection, including the concepts of SRG and the cost function of interest in the study. Based on the cost function, Section III introduces a novel matrix expression for deriving the sharable and non-sharable spare capacity along each link. Section IV presents the cost functions for solving the working and protection paths in a matrix form. Section V exposes the well studied special cases, namely protecting node and link failures. Section VI concludes the paper.

II BACKGROUND

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* This work has been done within the EU FP6 IP NOBEL (http://www.ist-nobel.org) framework.
A Notations

All scalar variables are printed with lower letters (e.g. $a$), all vectors are printed with underlined lower letters (e.g. $\underline{a}$), all sets are printed with underlined capital letters (e.g. $A$), and all two-dimensional matrices are printed with double underlined capital letters (e.g. $\underline{A}$). The links of the network are always directed representing a connection from one node to another.

B Concepts of Shared Risk Group (SRG)

Shared Risk Group (SRG) is defined as a group of network elements (i.e., links, nodes, physical devices, software/protocol identities, etc., or a mix of them) possibly subject to a common risk of a single failure. In practical cases, an SRG may contain several seemingly unrelated and arbitrarily selected network elements. We define that a working path is involved in an SRG if it traverses any network element that belongs to the SRG. Two working paths share the same risk of a single failure if they are involved in any common SRG. A working path is said to be SRG-disjoint with its protection path if the two paths are not involved in any common SRG. In this paper, the SRG-disjointness for a working and protection path-pair is the major effort of achieving 100% restorability for the working data flows under the single failure scenario.

Most of the past studies focused on the case where each single network element in the network topology serves as an SRG. However, when a general definition of the SRGs is desired, more complicated descriptions and further elaborations are required.

C The Input of the Shared Path Protection Problem

Given a graph $G(V,A)$ representing the network with a set of arcs $A$ and vertices $V$, where $|A|$ and $|V|$ are the number of links and nodes in the network. The cost for allocating a unit capacity on arc $j$ is denoted as $c_j \forall j \in A$. The unreserved free capacity along arc $j$ is denoted as $f_j \forall j \in A$. The capacity reserved for shared protection paths is called spare capacity denoted as $v_j \forall j \in A$ of each arc.

A common property of the graph representations proposed for single-layer single-domain networks [1-3], for multi-layer single-domain [4] and for single-layer multi-domain [5] networks, that each arc of the graph is assigned to a network element (e.g. optical node, a wavelength in the optical link). Thus, each SRG is composed of a set of network elements, and can be transformed as a set of links in the network. As a consequence, the SRGs are represented by a $|\text{SRG}| \times |A|$ matrix, denoted as $[\text{SRG}]$ (where $|\text{SRG}|$ denotes the number of total SRGs in the network). Furthermore, we are given the source node $s$ and the destination node $d$ and the bandwidth $b(W)$ of the new demand. Due to the complete routing information scheme, the full per-flow information of the network (i.e., the working and protection paths along each link) is present.

D The Goal of the Shared Path Protection Problem

The task is to find a working path (denoted as $W$) and the corresponding protection path (denoted as $P$) between the source $s$ and destination node $d$ with bandwidth $b(W)$. $W$ and $P$ should be SRG-disjoint. The feasible condition of the working path is $f_j \geq b(W)$ for $\forall j \in W$. With the presence of $W$, the spare capacity along link $j$ can be further categorized into (see also Fig. 3 for illustration): sharable spare capacity (denoted as $h^w_j$), which is the link capacity that has been reserved by some other protection paths, and is sharable to $P$; and non-sharable spare capacity (denoted as $s^w_j$), which is the link capacity that has been reserved by some other protection paths, and is not sharable with $P$ due to the SRG constraint. Note that $v_j = s^w_j + h^w_j$, which is the total spare capacity along link $j$.

![Fig. 3. An illustration for the capacity categories along a link.](image)

The feasible condition of the protection path is $f_j + v_j - s^w_j \geq b(W)$. The objective function will be defined in the next sections.

III THE MATRIX EXPRESSION FOR SHARED PROTECTION

A Generalisation of SRGs

The general SRGs can contain any combination of arcs of the graph. Thus the network can be turned into multiple isolated fragments when it is attacked by a failure of an SRG. Let us define a matrix $Z$ with size $|\text{SRG}| \times |V| \times (|V| - 1)$, where each column of the matrix corresponds to a node-pair and $z_{g_i,j}$ equals to 1 if the failure of the $g_i$-th SRG isolates node $i$ and $j$ and 0 otherwise. In other words it is 1 if the arcs of the $g_i$-th SRG form a cut between nodes $i$ and $j$. It is obvious that if a single-layer single-domain network is 2-connected and only link failures are protected, $Z$ is a 0 matrix.

It is clear that $|\text{SRG}|$ is one of the dominating factors for the complexity of signalling mechanisms and protocol computation. Therefore, a way of reducing $|\text{SRG}|$ under consideration without losing 100% restorability for any possible failure is critical to the development of the related mathematical formulation. It is trivial that if the $g_{r_i}$-th SRG is a part of the $g_{r_i}$-th SRG $(\text{SRG}^r_\beta \cdot e_{g_{r_i}} \leq \text{SRG}^r_\gamma \cdot e_{g_{r_i}}$, where $e_{g_{r_i}}$ is a unit column vector with a size of $|\text{SRG}^r|$ and if the failure of $g_{r_i}$ separates the nodes of the network into the same sets as the failure of $g_{r_i} (Z^r_\gamma \cdot e_{g_{r_i}} = Z^r_\beta \cdot e_{g_{r_i}})$, then $g_{r_i}$ does not need to be considered. Fig. 1 demonstrates an example, where the SRG’s circled with dotted lines are covered by a larger SRG, and can be eliminated.
It is clear that each link can be contained in multiple SRGs. Let us define a basic set contained in SRGs as BSS, which is a disjoint set of arcs. Since the set of BSS’s (denoted as $\mathbf{BSS}$) covers all arcs of SRGs, each entry in SRG can be expressed as the union of some BSS’s with the number of BSS (denoted as $|\mathbf{BSS}|$) being kept the minimal (see also Fig. 1). This can be simply calculated in linear time. Note that in most of the cases each BSS is an arc of the graph. However, in the multi-layer peer graph model [4], each wavelength of the optical link is represented by an arc. Thus, if the SRGs are defined for link cuts and some node failures, each BSS contains nothing but all the arcs assigned to an optical link. However, in the case of protecting only link failures in a single-domain and single-layer networks, a one-to-one mapping between each BSS and each SRG (or links) can be derived.

Fig. 1. The basic sets of SRGs (BSS) of the network on Fig. 1.

**B Sharable and Non-Sharable Spare Capacity Matrices**

This study summarizes the necessary steps of deriving $h_j^w$ for all $j \in L$, with the generalized definition of SRGs. The amount of sharable spare capacity along link $j$ demanded by $P$ (which is denoted by $h_j^w$) can be derived by performing matrix operations on the spare provision matrix [1,2]. However, the study in [1,2] can only deal with the case that each link is treated as an SRG (or called Shared Risk Link Group (SRLG)). To consider the general definition of SRGs, the mathematic formulation developed in [1,2] must be solidly expanded.

The spare provision matrix is denoted as $S'$, which is a $|A| \times |\mathbf{SRG}|$ matrix. The entry $(i, j)$ of $S'$ (denoted as $s_{i,j}$, where $i = 1 \ldots |\mathbf{SRG}|$, $j = 1 \ldots |A|$) keeps the amount of non-sharable spare capacity along link $j$ for the protection path if the corresponding working path is involved in the $i$-th SRG. The most straightforward way of obtaining the matrix of $S'$ is to simulate the failure of each SRG and measure the amount of restoration traffic on each link.

In order to evaluate the spare capacity required by the working capacity in each BSS, we need to define another matrix $S$ (with each entry denoted as $s_{i,j}$, where $i = 1 \ldots |\mathbf{BSS}|$, $j = 1 \ldots |A|$) with a size of $|A| \times |\mathbf{BSS}|$ by converting $S'$ using the following formula:

$$s_{i,j} = \max_{\forall(l_i, l_j) \in T} s'_{i,j,l}$$

(4)

In Eq. (4), we take the non-sharable spare capacity along link $j$ required by the working capacity on the $l$-th BSS as the maximum of the non-sharable spare capacity along link $j$ by the working capacity in the $l$-th BBS on the $i$-th SRG contained in the $i$-th SRG except the SRG's, which failure isolates $s$ and $d$. We ignore the SRGs of $z_{l(l,s,d)} = l$ because $W$ cannot be restored.

Let $D$ denote the set of working paths already in the network. The amount of non-sharable spare capacity along link $j$ provided that $W$ passes the $l$-th BBS can be formulated as:

$$s_{l,j} = \sum_{j \in D} b(Q) \cdot b_{i,j} \cdot a_{i,j}^l$$

(5)

where $Q$ represents the $q$-th working path of $D$, while $b_{i,j}$ and $a_{i,j}^l$ are two binary indicators defined as follows:

$$b_{i,j} = \begin{cases} 1 & \text{if the $q$-th protection path passes link} j \\ 0 & \text{otherwise} \end{cases}$$

(6)

$$a_{i,j}^l = \begin{cases} 1 & \forall i \text{ SRG having the $l$-th BBS} \ z_{l(i,s,d)} = 0 \text{ and } z_{l(i,e,d)} = 0 \\ 0 & \text{otherwise} \end{cases}$$

(7)

where $s(q)$ and $d(q)$ represents the source and destination node of $Q$, respectively. With Eq. (5) we can derive the amount of non-sharable spare capacity upon link $j$ required by all the working paths involved in the $l$-th BBS. It is the summation of bandwidth of all the restorable working paths on the $l$-th BBS because the working paths may be subject to a failure at the same moment. In this case, all the affected working paths can be restored provided that the source and destination nodes are not isolated due to the failure.

With the single failure scenario, only one SRG could possibly be subject to an interruption at a moment. Thus, we can derive $s_{l,j}^w$ for $\forall j \in L$, by finding the maximum demand of spare capacity among all the BSS’s traversed by $W$, i.e.,

$$s_{l,j}^w = \max_{l \text{ is taken by } W} s_{l,j}$$

Finally we can define sharable spare capacity matrix ($H$) with a size of $|A| \times |\mathbf{BSS}|$. We have known that the $(j,l)$ entry of $H$ stores the amount of non-sharable spare capacity along link $j$ by the protection path of $W$ provided that $W$ is involved in the $l$-th BSS. Thus, the $(j,l)$ entry in $H$ can be expressed as

$$h_{j,l} = v_j - s_{j,l}$$

(14)

We also define a matrix $R$ with the entry $(j,l)$ (i.e., $r_{j,l}$) standing for the ratio of the bandwidth required to be allocated as spare capacity along link $j$ if $W$ passes through link $l$ (similarly to 2).

$$r_{j,l} = \max \left\{ 0,1 - \frac{h_{j,l}}{b_{i,j}^l} \right\}$$

(15)

**IV FORMULATION OF COST FUNCTIONS IN MATRIX FORMS**

With the knowledge of $S$ (or $H$), the cost functions for both working and protection paths can be formulated in a matrix form. Let us define a column vector, denoted as $c_j$, for the cost along each link taken by $W$. $c_j$ is a vector of size $|A|$ where the $j$-th entry (denoted by $c_j$) is a non-negative value. Let us define a vector $w$ of size $|A|$ as the working path-
link incidence vector, where \( W = \sum_{i=1}^{n} e_i. \)

\( e_i = \{ 0, 1, 0, 0, ..., 0 \} \) depicts that \( W \) traverses through two links, namely the second and the \( x \)-th in the network. Let us define in a similar way the backup path-link incidence vector \( B \) with a size \( |1| \times |A| \), and a vector \( w' \) of size \( |1| \times |BSS| \) that stores the BSS’s of \( W \).

Let us define a matrix \( W \) representing \( W \), where \( W \) is a diagonal matrix of a size \( |A| \times |A| \) such that \( \text{diag}(W) = w \). In the same way, a matrix \( P \) is defined for \( P \) such that \( \text{diag}(P) = p \) and \( W' = \text{diag}(W') = w' \) and \( C \) as \( \text{diag}(C) = c \). We also define a special matrix norm similar to \( \| A \| = \max \sum_{i=1}^{n} |a_{i,j}| \), while the sum and max operation is swapped:

\[
\| A \| = \sum_{i=1}^{n} \max |a_{i,j}|
\]

where \( n \) is the number of rows of \( A \). This is also a valid matrix norm due to the followings [8]:

\[
\| A \| > 0 \text{ if } A \neq 0 \text{, and } \| 0 \| = 0
\]

\[
\| c \cdot A \| = |c| \| A \|
\]

\[
\| A + B \| \leq \| A \| + \| B \|
\]

\[
\| A - B \| \leq \| A \| - \| B \|
\]

Eq. (17) and Eq. (18) trivially hold, while Eq. (19) holds for the following reason:

\[
\| A + B \| = \sum_{i=1}^{n} \max |a_{i,j} + b_{i,j}|
\]

\[
\leq \sum_{i=1}^{n} \max |a_{i,j}| + \max |b_{i,j}|
\]

\[
\sum_{i=1}^{n} \max |a_{i,j}| + \sum_{i=1}^{n} \max |b_{i,j}| = \| A \| + \| B \|
\]

where the first \( \leq \) stands for the fact that \( |a_{i,j} + b_{i,j}| \leq |a_{i,j}| + |b_{i,j}| \), and the second is for the fact that \( \max(|a_{i,j}| + |b_{i,j}|) \leq \max |a_{i,j}| + \max |b_{i,j}| \). The proof of Eq. (20) is as follows:

\[
\| A - B \| = \sum_{i=1}^{n} \max \sum_{k=1}^{m} |a_{i,k} - b_{i,k}|
\]

\[
\leq \sum_{i=1}^{n} \max \sum_{k=1}^{m} |a_{i,k}| - \max |b_{i,k}|
\]

\[
\sum_{i=1}^{n} \max |a_{i,j}| - \sum_{i=1}^{n} \max |b_{i,j}| = \| A \| - \| B \|
\]

where \( m \) is the number of rows of \( B \). The first \( \leq \) stands for the fact that \( |a_{i,j} - b_{i,j}| \leq |a_{i,j}| \), and the second is for the fact that \( \max \sum_{k=1}^{m} |b_{i,k}| \leq \sum_{k=1}^{m} \max |b_{i,k}| \).}

### Cost of the working path

The cost of allocating \( W \) can be derived by summing up the cost along each link taken by \( W \):

\[
c(W) = b(P) \cdot \| C^T \cdot W \|
\]

(22)

Since \( C \) is a diagonal matrix, we have \( C^T = C \). Eq. (22) holds because \( C^T \cdot W \) (with a size \( |A| \times |A| \)) can be expressed as \( \sum_{i=1}^{n} c_{i,j} \cdot b_{i,j} \) and \( c_{i,k} = 0 \) if \( i \neq k \). Thus, \( C^T \cdot W \) is nothing but a diagonal matrix with each entry \( c_{i,j} \cdot b_{i,j} \) for \( i = 1 \) to \( |A| \). The norm of the matrix can be expressed as \( \| C^T \cdot W \| = \max \sum_{i,j} |c_{i,j} \cdot b_{i,j} | = \sum_{i,j} |c_{i,j} \cdot b_{i,j} | \), which is nothing but the total cost of \( W \) if we multiply it by the bandwidth of the connection (\( b(W) \)) as was defined at Eq. (1).

### Cost of the Protection Path

The total cost of \( P \) can be expressed as:

\[
c(P) = b(P) \cdot \| C^T \cdot P^T \cdot R \cdot W' \|
\]

(23)

since \( P \) is a diagonal matrix, we have \( P^T = P \). Eq. (23) holds since \( C^T \cdot P^T \cdot R \cdot W' \) is a matrix with the \( (i,j) \) entry as \( \sum_{k=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot p_{i,k} \cdot r_{k,l} \cdot w_{l,j} \). It is multiplied by a diagonal matrix \( C^T \) from the left, which is equivalent to the case that each row of the matrix is multiplied with \( c_{i,j} \). Thus, the \( (i,j) \) entry in \( C^T \cdot P^T \cdot R \cdot W' \) is \( \sum_{k=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot p_{i,k} \cdot r_{k,l} \cdot w_{l,j} \), which is a matrix with each entry \( c_{i,j} \cdot r_{k,l} \) as \( 1 \) if the \( k \)-th link is a part of \( P \) and the \( l \)-th BSS is traversed by \( W \), and 0 otherwise. The norm of this matrix equals to

\[
\| C \cdot P^T \cdot R \cdot W' \| = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot p_{i,k} \cdot r_{k,l} \cdot w_{l,j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \max \sum_{k=1}^{n} c_{i,j} \cdot p_{i,k} \cdot r_{k,l} \cdot w_{l,j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \max \sum_{k=1}^{n} c_{i,j} \cdot p_{i,k} \cdot r_{k,l} \cdot w_{l,j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot r_{k,l} \cdot w_{l,j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot r_{l,j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{n} c_{i,j} \cdot r_{l,j} \]

where the second equality is true due to the fact that \( P \) is diagonal (i.e., \( p_{i,k} = 0 \) if \( i \neq k \)) and the third equality is true since \( W' \) is diagonal (i.e., \( w'_{l,j} = 0 \) if \( l \neq j \)). Finally the last equality in the above equation can be verified by observing Eq. (7). After multiplying the norm by \( b(W) \), we get the same cost function defined in Eq. (3).

Finally, the objective function can be expressed as:

\[
c_{\text{total}} = c(W) + c(P) = b(W) \cdot \left( \| C^T \cdot W \| + \| C^T \cdot P^T \cdot R \cdot W' \| \right)
\]

(24)

The least-cost diverse routing for shared protection can be conducted based on the matrix operations defined above and the cost function in Eq. (24).

### V A CASE STUDY

This section investigates the case that an SRG is defined as either a node or a link; in other words, both node-failure and link-failure are considered. In this special case, the directed graph representing the network is built using graph
transformation technique called node-splitting [7], where two twin vertices represent a protected node. For each pair of twin vertices, one of the vertices is the incoming vertex and the other is the outgoing vertex. The incoming arcs of the node are connected to the incoming vertex, and all the rest are connected to the outgoing vertex. An additional arc is inserted between each pair of twin nodes, directing from the incoming vertex to the outgoing vertex. The source node of the demand is split in the graph, where $s$ is assigned to the outgoing vertex; similarly, the destination node is split, too, where $d$ must be assigned to the incoming vertex.

Using the node-splitting approach, we get a special graph structure, where there is a one-to-one mapping between each pair of SRG and arc. The only thing that needs to be verified is that an SRG for a node failure can be assigned to the additional arc between the incoming and the outgoing vertex. This comes from the fact that only if a path passes through the additional arc of a vertex in the graph, will it pass through the node.

$S'$ has a size of $|A| \times (|A| + |N|)$ since there are SRGs of link failure and SRGs of node failure. The matrix $S'$ has a size of $|A| \times |A|$ since BSSs are the links of the network.

### A Simulation Results

An experiment is conducted to analyze the surplus network resources consumed to handle all the single failure cases where both node-failure and link-failure are considered. The case where only the link-failure cases are considered is taken for comparison. Two networks with 16 and 30 nodes are adopted in the experiment, which are first randomly loaded with light, mid and high traffic at the beginning.

Fig. 5. The 16-node (n16) and 30-node (n30) network.

An ILP formulation for SPP that can yield an optimal solution is adopted [6] and solved by CPLEX, where each network topology has 400 connection requests being launched. A statistics was generated on the average cost of the connection (the allocated capacity multiplied with the edge costs) and the average length of the working paths. The case where only the link-failures are considered is taken for comparison. The surplus is shown on Fig. 6 versus traffic load (in Erlang). It is clear that in networks with many unreserved capacity, 4% of extra bandwidth need to be allocated in case of protecting both link and node failures compared with the case where only link failures are considered, while in highly loaded networks, the extra bandwidth demand is 7-9%.

![Fig. 6(a)-(b). The average surplus cost versus traffic load in case of protecting both node and link failures compared with the case with only link failures on the 16- and 30- node network, respectively.](image)

**CONCLUSIONS**

This paper has demonstrated a compact mathematical formulation in solving the problem of survivable routing for shared protection in communication networks with bandwidth guaranteed tunnels. The proposed formulation takes the most general definition of Shared Risk Groups (SRGs) that can consider both link- and node-protection. Based on the general definition of SRGs, the developed formulation can flexibly solve the problem of survivable routing for shared path protection on any network topologies and any graph models developed, including graph models of multilayer peer models or hierarchical multi-domain graph structures. A unified expression of the cost functions for both working and protection paths can be derived under the complete routing information scenario. We conclude that the generalised matrix expression for solving the dynamic survivable routing problem can significantly facilitate the development of shared protection schemes for various applications.

**REFERENCES**


