Abstract—Protection techniques for optical networks mainly rely on pre-allocated backup bandwidth, which may not be able to provide full protection guarantee when multiple failures occur in a network. To protect against multiple concurrent potential failures and to utilize the available resources more efficiently, strategies such as backup reprovisioning rearrange backups of protected connections after one failure occurs or, more generally, whenever the network state changes, e.g., when a new request arrives or terminates. Recently, new solutions for automated management in Optical networks promise to allow customers to specify on-demand the terms of the Service Level Agreement (SLA) to be guaranteed by the service provider. In this paper we show that is possible to further reduce the capacity requirements of backup reprovisioning techniques exploiting the knowledge, among the other Service Level Specifications (SLS), of the connection holding-time.

Our main contributions are as follows. First, we prove that the problem of backup reprovisioning for all the lightpaths requiring shared-path protection under a current network state is NP-complete. Second, we provide a mathematical ILP model of the reprovisioning problem considering the additional holding-time information. Third, since the problem is NP-complete and we cannot efficiently rely on exact approaches, a new global reprovisioning algorithm called Ph-GBR is proposed which can significantly reduce the resource overbuild exploiting the information about connection durations. By means of simulative experiments, we compare capacity requirement and computational complexity of Ph-GBR to those of another holding-time unaware, yet efficient algorithm, called GBR, considering a dynamic traffic in a wavelength-convertible WDM mesh network scenario.

Index Terms—Optical network, WDM, NP-complete, dynamic traffic, holding time, shared protection, reprovisioning, ILP

I. INTRODUCTION

Optical networks provide a transport infrastructure with very high capacity, thanks to wavelength-division-multiplexing (WDM) technology. The huge bandwidth of WDM also requires efficient survivability mechanisms, because the failure of a network element (usually a node or a link) can cause a large amount of data loss; a highly available WDM layer is crucial to enable Quality-of-Service sensitive applications over it. Many existing transport networks use SONET rings, but they typically rely on excessive capacity redundancy.

Recently, various protection and restoration mechanisms have been proposed to efficiently deal with this problem in mesh networks [1]. Among them, the backup reprovisioning (BR) represents a recent and valuable candidate for path protection. In path protection schemes, whenever a connection is provisioned, extra bandwidth is reserved so that the connection is served by a primary and backup path pair. BR consists in a re-arrangement of the backup capacity during network operation that is either periodical or triggered by network-status changes, such as connection provisioning or failures. This new procedure promises to effectively address two key requirements for protection strategy: first, a high (and desirable) resource efficiency and, second, the capability to recover from multiple failures.

Recently, in order to efficiently accommodate the backup capacity, new challenges, but also opportunities, are offered by network evolution of the prevalently static traffic towards a more dynamic traffic paradigm. As a matter of fact, so far optical transport networks have been supporting connections, which are provided and leased for long period of time, e.g. weeks or months. Nowadays, many new applications are emerging with requirements of large bandwidth over relatively short periods of time: let us consider for example video distribution of important sport or social event, or the massive data transfer for backup or storage purposes.

Technology and bandwidth market are developing to provide the flexible platform the new applications are asking for. New agile optical crossconnects (OXC) are emerging to create mesh-structured optical WDM backbone networks and, in order to manage and control dynamic WDM networks, new control protocols have been proposed; ASON/ASTN and GMPLS are protocol-independent, control-plane architectures (standardized by ITU and IETF, respectively). New architectures and routines for user-controlled on-demand optical circuit provisioning [2] based on automatic or web-based interfaces at the the management plane (MP) [3] will enable the on-line specification of the SLA terms to be guaranteed (with different price range) by the service provider1. More specifically, we consider that these service level specifications (SLS) coming from the MP could be exploited by the control plane to improve the efficiency of the routing algorithms: among the other SLSs, we propose to exploit the knowledge of the connection-holding time to efficiently solve the backup reprovisioning (BR) problem.

In this paper we provide, to the best of our knowledge for the first time, a rigorous NP-completeness proof of the BR

1Note that both ASON and GMPLS use distributed real-time signaling that, in conjunction with appropriately extended versions of OSPF-TE, can take charge of the distribution of this information coming from the MP.
problem. Then, to clearly state the BR problem with holding-time knowledge, we present an analytical formulation based on Integer Linear Programming of the reprovisioning problem in its holding-time-aware fashion. Finally, since the problem is NP-complete and ILP formulations are unapplicable on large network instances, we propose a new holding-time-aware heuristic approach for backup reprovisioning. The opportunity to exploit the connection durations has been already exploited for shared-based protection strategies in [4] achieving significant improvements in backup-channel capacity utilization. We compare our approach to Generalized Backup Reprovisioning (GBR) [5], which have been shown to be very capacity efficient and allows the protection of multiple failures, but it is holding-time unaware. For a typical US nationwide network, we obtained savings on resource overbuild of up to 10% for various practical scenarios.

The rest of this paper is organized as follows. Section II overviews the background work on backup reprovisioning. In Section III we discuss some fundamental issues about the interaction of shared-path protection (provisioning phase) and backup reprovisioning (reprovisioning phase). In Section IV we formally state the BR problem and we analyze BR complexity (deferring the the NP-completeness proof details in the Appendix). In Section V we introduce a known approach for BR, called GBR. Section VI provides the ILP formulations that models the holding-time aware BR problem and a new connection-holding-time-aware procedure is discussed: this new methodology, named as PHOTO (Provisioning by HOlding Time Opportunity) is applied to the standard algorithm GBR, giving origin to the new algorithm Ph-GBR. Section VII evaluates by simulations the performance of Ph-GBR compared to the GBR algorithm. Section VIII draws some conclusions.

II. PRIOR WORK

Lightpath reprovisioning or re-optimization is a proactive procedure in which network capacity is re-arranged and reprovisioned for existing connections during network operation. More generally, a scheme that reprovisions both primaries and backups [6] could achieve better capacity optimization compared to a scheme that reprovisions only backups, but the former may also result in service interruptions since some primary paths may be rerouted. Moreover, in [7] the results indicate that most of capacity savings can be achieved by only employing backup reprovisioning. Therefore, backup reprovisioning is still attractive to network operators for its capacity efficiency. In particular, it has been shown in [6] that shared-path protection with reprovisioning can provide the same level of robustness compared to dedicated-path protection while utilizing significantly less capacity. Most works focus on the shared-path-protection strategy with reprovisioning, which is applied in our study as well.

In [8] backup paths are reassigned from a precomputed path set to optimize the usage of backup capacity. Authors in [9] present a backup relocation policy to accommodate requests that would otherwise have been rejected due to limited usage of wavelength converters. In [10], a migration scheme has been proposed where backup paths are migrated to paths selected from a set of \( k \) precomputed paths. The lightpath re-optimization problem has been formulated as an integer linear program (ILP) in [11] [12].

In [13] a linear program (LP) formulation is presented, to provide a lower bound on the required backup resources.

Variety of works referred to pre-emptive reprovisioning schemes also to handle multiple "concurrent" failures [6], [14], [15], [16], [17]. In [5] backup reprovisioning is performed either to affected connections which lose their primary or backup due to the previous failures or due to backup sharing (MBR), or to all existing connections (GBR). In order to reprovision fewer connections, the work in [18] proposes an algorithm which only reprovisions connections that lose their primary paths with new one after the failure. In [19] it has been show that it is desirable to perform reprovisioning after a failure is repaired as well as after failure arrival.

III. BACKUP REPROVISIONING AND SHARED PATH PROTECTION

The backup reprovisioning works as an accessory protection procedure which allows us to re-arrange the backup capacity during network operation. As mentioned in the previous section, this procedure (reprovisioning phase) has to be applied differently in accordance to the protection strategy that is enforced during normal network operation (provisioning phase).

In this work, we assume a shared-path protection (SPP) provisioning scheme. We recall that, under this assumption, the working and backup paths \( l_w \) and \( l_b \) of a new incoming connection must satisfy the following constraints with respect to the other existing lightpaths \( l^i_w \) and \( l^i_b \) as follows:

- \( C.1 \) \( l_w \) and \( l_b \) are link disjoint.
- \( C.2 \) \( l_w \) and \( l^i_w \) do not utilize the same wavelength on any common link they traverse.
- \( C.3 \) \( l_w \) does not share any wavelength with \( l^i_b \) on any common link they traverse.
- \( C.4 \) \( l_b \) and \( l^i_b \) can share a wavelength on a common link only if \( l_w \) and \( l^i_w \) are link disjoint.

Many effective strategies to dynamically provision shared-path-protected connections have been proposed in recent years (see [20] for a detailed list of references). Since the aim of this work is to increase the capacity efficiency taking profit of the knowledge of connection durations, we resort to two SPP alternative strategies, a holding-time-unaware approach (CAFES [20]) and its holding-time-aware counterpart PHOTO [4]. For sake of brevity, in the following we will only briefly describe the two previous SPP-routing approaches, while more details on the role played by the connection-holding-time in the reprovisioning phase will be given in the rest of the paper.

CAFES [20] computes a feasible solution (i.e., two link-disjoint paths) based on a two-step approach: first, a set of \( K \) candidate working paths are computed; then, by means of a opportune cost-assignment strategy, the associate backup paths are selected in order to minimize the resource utilization...
(by increasing backup sharing) and meet the shared-path-protection constraints C.1-C.4. Among the \( K \) candidate path pairs the one with the lowest cost is chosen.

PHOTO represents the extension of the previous algorithm to effectively include holding-time knowledge. It is still a two-step method: a set of \( K \) admissible working paths is identified, and a respective set of backup paths is evaluated based on link cost redefinition. But, in this second case, a new holding-time aware cost assignment will estimate the amount of time during which the link is sharable along the connection lifetime (while the CAFES’s cost assignment expresses only if a link is sharable or not at the instant of arrival of the new connection), so that PHOTO is able to capture the additional information provided by holding-time knowledge obtaining a remarkable gain on resource usage.

Finally, it should be noted that the backup re-arrangement can be carried on periodically, e.g. beyond a given threshold of routed connections or after a given amount of time, or it can be triggered by network-status changes such as connection provisioning or failures. For sake of terseness, in the following we focus on the case of reprovisioning for backup resources optimization, neglecting the case of reprovisioning after a link failure. Details about the needed changes in this former case can be found in [5].

IV. PROBLEM STATEMENT AND COMPLEXITY ANALYSIS FOR BACKUP REPROVISIONING

We first define the notations and then formally state the dynamic, shared-path-protected holding-time-aware backup reprovisioning problem. A network is represented as a weighted, directed graph \( G = (V, E, C, \lambda) \), where \( V \) is the set of nodes, \( E \) is the set of unidirectional fibers (referred to as links), \( C : E \rightarrow \mathbb{R}^+ \) is a function that maps the elements in \( E \) to positive real numbers representing the link costs, and \( \lambda : E \rightarrow \mathbb{Z}^+ \) specifies the number of wavelengths on each link (where \( Z^+ \) denotes the set of positive integers).

We use \( \lambda^*_e \) to denote the number of free wavelengths on link \( e \in E \). We denote the set of existing lightpaths in the network at any time by \( L = \{\{\ell^w_1, t^w_1, t^b_1, t^b_1\}\} \), where the quadruple \( \{\ell^w_1, t^w_1, t^b_1, t^b_1\} \) specifies the working path, the backup path, the arrival time and the holding time for the \( i \)th lightpath. In particular, we denote the set of existing working path as \( L_w \) and the set of the corresponding backup paths as \( L_b \).

We associate a link vector [20] with each link in the network, to identify the sharing potential between backup paths. The link vector \( \nu_e \) for link \( e \) can be represented as an integer set, \( \{\nu^e_0 | e \in E, 0 \leq \nu^e_0 \leq \lambda^i(e)\} \), where \( E \) is the set of links; \( \lambda^i(e) \) specifies the number of wavelengths on link \( e^i \); and \( \nu^e_0 \) specifies the number of working lightpaths that traverse link \( e^i \) and are protected by link \( e \) (i.e., their corresponding backup lightpaths traverse link \( e \)). Through such a simple data structure, the link vector captures the necessary information on the sharing potential offered by each link. The number of wavelengths which need to be reserved for backup lightpaths on link \( e \) is thus \( \nu^w_e = \max_{e^i \in E} \{\nu^e_0 \} \). Therefore, using the link vector, we can simply reserve \( \nu^w_e \) wavelengths on link \( e \) as backup wavelengths.

At the instant \( T \) all the backups in the network are reallocated, either to face a link failure or to re-optimize the backup capacity distribution.

We now formally state the dynamic shared-path-protected backup-reprovisioning problem as follows: given a WDM network as \( G = (V, E, C, \lambda) \) and the set of existing lightpaths (or the associated conflict sets \( \{\nu^w_e | e \in E\} \)), whose backup paths have to be reprovisioned at instant \( T \), find a path assignment for all the backup paths under the shared-path constraints C.1-C.4, minimizing the amount of requested backup capacity\(^2\).

A. Complexity Analysis

We formally state the decision version of the dynamic shared-path-protected backup-reprovisioning (SPP-BR) problem below and prove that it is NP-complete.

Instance: A graph \( G = (V, E, C, \lambda) \), the set of existing working lightpaths \( L_w \), and the corresponding backup lightpath requests.

Question: Does there exist a set of \( L_b \) backup paths, each for existing connection (then \( |L_b| = |L_w| \)), that satisfy the shared-path-protection constraints and that is composed of no more than \( B \) backup wavelengths?

Theorem 1: SPP-BR is NP-complete.

Proof: Please refer to Appendix.

V. GBR: AN EFFICIENT ALGORITHM FOR SPP-BR WITHOUT HOLDING-TIME AWARENESS

Since the BR problem is NP-complete, we have to resort to heuristics. In the following we provide a complete description of the GBR heuristic proposed in [5]: then, in the next section, we will describe incrementally the modifications needed to incorporate the holding-time knowledge in the GBR approach.

A formal specification of the heuristic is given in Algorithm 1. In this heuristic, given the list \( L \) of connections in the network, the \( |L| \) connections are ordered according to \( S = \{S_1, \ldots, S_{|S|}\} \) different sequences, which are randomly generated. Let us consider a single sequence, e.g. \( S_j \): it represents a reprovisioning attempt in which all the backup paths are reprovisioned using a shortest-path algorithm. The cost of each reprovisioning attempt \( C_{S_j} \) is evaluated in terms of backup capacity (number of wavelengths). Among all the \( S \) sequences, the solution with the minimum backup capacity consumption is selected.

When routing each of backup paths, a key-role is played by the link-cost assignment in Step 3. Since the working paths are fixed, GBR sets an appropriate link-cost assignment to route the corresponding backup paths while minimizing the backup capacity. Using the information stored in the link vector, while reprovisioning the \( i \)th connection of a generic \( S_j \) sequence, we retrieve the information about the amount of

\(^2\)A possible alternative objective function consists in minimizing the number of unprotected connections, but, since this last case better fits the backup reprovisioning after a link failure, from now on we only consider the first option.
Algorithm 1 GBR Reprovisioning Algorithm [5]

Input: $G = (V, E, C, \lambda)$, $\nu = \{\nu_e \mid e \in E\}$, $L = \{(l_w^i, l_b^i, t^i_a, t^i_h)\}$.  
Output: A new set of backup paths $\{l_b^i\}$ satisfying shared-path-protection constraints, with minimal amount of additional capacity.

Costs: $C_{Repr}$ (minimal cost for reprovisioning), $C_{S_j}$ (cost of the $j$-th reprovisioning attempt $S_j$).

1) Release the backup resources for all the connections in the network, $C_{Repr} = \infty$, $j = 1$  
2) Generate a random-ordered list $S_j$ of the connections in $L$ and set the cost $C_{S_j} = 0$, $i = 1$  
3) Reprovision a new backup for the $i$-th connection in $S_j$.  
When computing the backup route, define a the cost of link $e$, $\forall e \in E$, as follows:

$$C_b(e) := \begin{cases} +\infty & \text{if } e \in l_w^i \lor (\lambda^f_e = 0 \land \exists e' \in l_w^i | \nu^e_e' = \nu^e_e) \\ \epsilon & \text{if } \forall e' \in l_w^i, \nu^e_e' < \nu^e_e \\ 1 & \text{otherwise} \end{cases}$$

Apply the shortest-path algorithm to compute the backup route using the new link cost. Update link link vector and $\nu^*_e$ accordingly if a new backup can be found and allocate a new backup wavelength if needed; if $l_b^i$ is not found, $j = j + 1$, go the step 5; otherwise $i = i + 1$.

4) If $i \leq |S_j|$ go to step 3, otherwise compute $C_{S_j}, C_{Repr} = \min(C_{Repr}, C_{S_j})$ and $j = j + 1$.
5) If $j \leq |S|$, go to step 2, otherwise return the set of backup paths $L_b$ associated with the minimal cost $C_{Repr}$.

backups capacity already reprovisioned and allocated on link $e$ when routing the $i$-th backup. If the entire backup capacity is already used to protect the failure of a link along the respective working path (i.e., $\exists e' \in l_w^i | \nu^e_e' = \nu^e_e$), then the link has to be considered as not shareable. In this case, if there is free capacity $(\lambda^f_e > 0)$, then a new wavelength can be allocated and link cost is the full cost 1. Otherwise, if $(\lambda^f_e = 0)$, the link is not usable and its cost is set to infinite. Finally, if there are backup channels other than those reserved to protect a link along the working path (condition expressed by “if $\forall e' \in l_w^i, \nu_e^e' < \nu_e^e$”), then the link is shareable and we don’t allocate a new channel on it. So, the cost is set to a low value corresponding to a small constant $\epsilon$ (the utilization of $\epsilon$ prevents backup paths from being unnecessarily long).

VI. PROVISIONING BY HOLDING-TIME OPPORTUNITY (PHOTO) METHODOLOGY: THE PH-GBR APPROACH

Actually, the state of a given link changes during the incoming connections holding time when some existing connections depart and future connections arrive. For example, a link, which is considered as shareable at the instant of reprovisioning of a connection, may, during the lifetime of the connection, assume a different state due to the deallocation of backup capacity because of connection departures or due to allocation of new shareable capacity for new arrivals.

In this study, we have assumed that information on future connections may not be known in advance (this further information would lead to a static scheduling problem). Nevertheless, we could exploit at least the information about the departure events, which is simply retrievable from the knowledge of the connection-holding time. So, we could modify the link-cost assignment for backup paths evaluation to capture the future degree of “shareability” of a given link.

For the subsequent formalization, let us suppose, without loss of generality, that the backup reprovisioning is carried on a periodical basis, e.g. each $N$ requested connections. If the arrival rate is set to value $\rho$, the average time interval $I_{rep}$ between two reprovisioning events is given by $I_{rep} = N/\rho$.

In our approach, the connection residual time of interest is bounded by $I_{rep}$, because after this interval another event of reprovisioning will change the backup capacity distribution. So the connection residual time for a generic $i$-th connection is given by:

$$h_i := \begin{cases} I_{rep} & \text{if } (t^i_a + t^i_h > T + I_{rep}) \\ t^i_a + t^i_h - T & \text{otherwise} \end{cases} (1)$$

A. Time-Enhanced Link Vector

In order to follow step-by-step the changes in the link states, we have to upgrade the link vector descriptor to follow the capacity distribution over the network’s link along the connection lifetime. So, we introduce the new symbols $\nu^e(\Delta \tau_k), \nu^e(\Delta \tau_k)$ and $C_b(e, \Delta \tau_k)$, which express the values of $\nu^e, \nu^e(\Delta \tau_k)$ and $C_b(e)$, respectively, in the interval of time $\Delta \tau_k$.

Let us expressly define $\Delta \tau_k$ first. According to values returned by Eq. 1, the $h_i$’s can be ordered so that $h_i \leq h_{i+1}$, $i = 1, 2, ..., L$. As a consequence, $\tau = \{\tau_0, ..., \tau_L\} = \{0, h_1, h_2, ..., h_L\}$ will indicate the departure events before the next reprovisioning event at instant $T + I_{rep}$ and $\Delta \tau_k = \tau_k - \tau_{k-1}$ expresses the time interval between two departures. Link vector $\nu^e(\Delta \tau_k), \nu^e(\Delta \tau_k)$ and associated cost $C_b(e, \Delta \tau_k)$ will be updated according to the $k$-th connection departure. In other words, we have divided the interval $I_{Rep}$ into a series of time intervals $\Delta \tau$ which expresses the distance between two departures.

B. Mathematical formulation

An ILP formulation for the holding-time-unaware version of the problem can be found in [5]. In the following, we report the mathematical formulation of the holding-time-aware SPP-BR problem.

Let us define all the variables involved in this protected flow formulation:

- $x^b_{ij}$ is a boolean variable indicating whether the backup path of connection $l$ crosses the link $(i, j)$  
- $K_{ij}(\Delta \tau_k)$ is an integer variable expressing the number of backup wavelengths to be reserved on link $(i, j)$ during the time interval $\Delta \tau_k$.

The following additional symbols are also defined:

- $F_{ij}$ is number of free wavelengths on link $(i, j)$
• $R_{ij}^{lw}$ keeps track of the primary path of connection $l$: $R_{ij}^{lw}$ is a boolean variable indicating whether the primary path of connection $l$ crosses the link $(i, j)$.
• $s_l$ ($d_l$) is the source (destination) node of connection $l$.
• $Y_l(\Delta \tau_k)$ expresses the existence of the connection $l$ in the time interval $\Delta \tau_k$.

Now we can detail the flow-based formulation. The cost function to be minimized is total capacity allocated in time to serve all the backup paths, while the second term (multiplied by a small constant $\alpha$) is used to avoid loop and unnecessarily long paths.

$$\min \sum \Delta \tau_k \left[ \sum_{(i,j) \in E} K_{ij}(\Delta \tau_k) + \alpha \sum_{l \in L} \sum_{(i,j) \in E} x_{ij}^{lb} \right]$$

The set of constraints is the following

$$\sum x_{kj}^{lb} - \sum x_{jk}^{lb} = \begin{cases} -1 & \text{if } j = s_l \quad \forall j \in V, \\ 1 & \text{if } j = d_l \quad \forall l \in L; \\ 0 & \text{otherwise} \end{cases} \quad \forall (i,j) \in E;$$

$$R_{ij}^{lw} + x_{ij}^{lb} + x_{ij}^{lb} \leq 1 \quad \forall (i,j) \in E, l \in L;$$

$$K_{ij}(\Delta \tau_k) \leq F_{ij} \quad \forall (i,j) \in E, \Delta \tau_k;$$

$$\sum_{l \in L} R_{ij}^{lw} x_{ij}^{lb} Y_l(\Delta \tau_k) \leq K_{ij}(\Delta \tau_k) \quad \forall (i,j), (i',j') \in E, (i,j) \neq (i',j'), \Delta \tau_k;$$

This formulation assigns for each backup path a routing with respect to shared-path-protection strategy, enforcing constraints C.1-C.4. Constraint (2) is a solenoidality constraint, which assures that the backup paths are routed between their end-nodes. Constraint (3) enforces the link-disjointness between the primary and the backup path. Constraint (4) ensures that the total number of WDM channels allocated for spare paths on the unidirectional link $(i, j)$ is not larger than the available capacity, given by the number of wavelength $F_{ij}$. Finally, by Constraint (5) we can enforce that the number of connections, whose primary path crosses link $(i',j')$ and whose backup path is on on link $(i,j)$, must be less or equal to the number of backup wavelengths reserved on link $(i,j)$: the input values $R_{ij}^{lw}$ and $Y_l(\Delta \tau_k)$ and the variable $x_{ij}^{lb}$ can assume only value 0 or 1; only when they are all unitary, their product will not be zero. Note that constraints 3, 4 and 5 are verified during each of the time intervals considered in our holding-time aware approach (i.e. the constraint is repeated for each $\Delta \tau_k$). The previous model has $|E|(|L|+N)$ variables and $|L|(|V|+|E|)+|E|^2N$ constraints.

**C. PHOTO Link-Cost Assignment**

By introducing holding-time-awareness in a routing algorithm and the associated time-enhanced link vector, the usual assignment of link costs can be improved. Our proposal consists of minimizing not only the (current) additional capacity, but also the cost of additional wavelengths multiplied by the estimated time interval during which this additional wavelength has been provisioned for the reprovisioned connection. This cost assignment will improve the resource overbuild associated with backup paths, because the optimization metric has been evolved from wavelength mileage to wavelength mileage times time.

In other words, we divide the time interval $I_{rep}$ into a series of time intervals $\Delta \tau$ which express the distance between two departures. The cost of link $e$ during the interval $\Delta \tau_k$ will be re-evaluated after having considered the departure of the $k$-th connection until $T + I_{rep}$, according to the following scheme:

$$C_b(e, \Delta \tau_k) := \begin{cases} +\infty & \text{if } e \in I_{rep}^k \lor (N_e = 0 \land \exists e' \in I_{rep}^k, \nu_e^c(\Delta \tau_k) < \nu_e^c(\Delta \tau_k)) \\ \epsilon & \text{otherwise} \end{cases}$$

It is worth noting that 1) all of the previously-cited parameters are evaluated at each reprovisioning instant, 2) future arrivals are not known, and 3) a generic connection $i$ ignores the departure events after a time equal to $\min(T + I_{rep}; t_a + t_r)$. Overall the link costs $C_b(e)$ for each connection $i$ will be evaluated by considering each connection departure and by summing the cost contribution due to any time interval $\{\Delta \tau_k | k \leq i\}$ in the following manner:

$$C_b(e) = \frac{1}{h_i} \sum_{k=1}^{\Delta \tau_k} (\Delta \tau_k) \cdot C_b(e, \Delta \tau_k) \quad e \in E$$

The new algorithm obtained by replacing the old holding-time unaware link-cost assignment in the step 3 of Alg. 1 with the new PHOTO link-cost assignment in the GBR algorithm will be referred from now on as Ph-GBR. The new cost function to be minimized is not only the wavelength utilization at the reprovisioning instant, but a more meaningful estimation, which considers the wavelength usage along the entire inter-reprovisioning period.

The computational complexity of the heuristic Ph-GBR is $O(S|L|N|E|^2)$, mainly given by the algorithm step 3, whose complexity is $O(N|E|^2) + O(|E| + |V| \log |V|) = O(\max\{|N| |E|^2, |E| + |V| \log |V|\}) = O(N |E|^2)$, due to the procedure of cost assignment and Dijkstra’s algorithm with Fibonacci heap, respectively.

**D. A wiser Choice**

In Fig. 1, we show the state of a simple network during the application of the reprovisioning at the instant, e.g., $T = 10$ and subjected to an mean interval of reprovisioning $I_{rep} = 50$. The arrival instant and the holding time of the three connections present in the network are $t_a^1 = t_a^2 = t_a^3 = 0$ and $t_b^1 = 30, t_b^2 = 20, t_b^3 = 40$ respectively. Then at time $T = 10$ the remaining holding times for the connections are $h_1 = 20, h_2 = 10, h_3 = 30$. If the $h_i$ of a generic connection was larger than $I_{rep}$, we should have set $h_i = I_{rep}$. Suppose, during one of the $S_j$ reprovisioning attempts, that the Ph-GBR have already reprovisioned the backup paths of the two connections $r_1$ and $r_2$ like in Fig. 1, but not yet $r_3$. Clearly, $h_1 > h_2$ and the routing of the $r_3$’s backup path along links used by the backup of $r_1$ would lead to longer period of backup resource
sharing than choosing a path along the links used by \( r_2 \)'s backup path.

In accordance to GBR’s link cost assignment, \( r_3 \)'s backup path could be routed on two equivalent paths (see the arrows in the Fig. 1): let us call them upper path (along nodes E-C-D-F) and lower path (along nodes E-G-H-F). The cost is equal to \( 2 + \epsilon \) in both cases. But, we can observe that the upper path on link C-D will share backup capacity with connection \( r_1 \) for a longer time than the lower path on link G-H with connection \( r_2 \). Since \( h_3 - h_2 > h_3 - h_1 \) the lower path requires additional capacity for the time duration \((h_1 - h_2)\) while the upper path does not require the additional capacity during this time interval. Applying Ph-GBR link cost assignment, then the total cost to route \( r_3 \)'s backup on the upper path will be \( C_{tot} = (20\epsilon + 10)/30 + 2 \) instead of \( C_{tot} = (10\epsilon + 20)/30 + 2 \).

Fig. 1. In this simple network example, GBR link-cost assignment does not allow to identify which of the two alternatives would be the likely more convenient choice. Holding-time-aware Ph-GBR succeeds instead.

VII. RESULTS

We now quantitatively evaluate the performance of our proposed Ph-GBR algorithm compared to the baseline GBR algorithms. We simulate a dynamic network environment with the assumptions that the connection-arrival process is Poisson and the connection-holding time follows a negative exponential distribution. For the illustrative results shown here, in every experiment, \( 10^5 \) connection requests are simulated. Requests are uniformly distributed among all node pairs; average connection-holding time is normalized to unity; the cost of any link is unity; the example network topology with 16 wavelengths per fiber is shown in Fig. 2.

We employ two metrics to highlight the performance improvement achievable by PHOTO application: total channel consumption and resource overbuild. In all our scenario blocking probability is null or very small to allow a fair comparison of the two strategies, focused on the capacity requirements.

Two main parameters are varied in the following analysis: \( N \), the number of connections that are provided between two reprovisioning events, and \( K \), the number of alternate working paths that are explored by CAFES and PHOTO during the provisioning phase to avoid trap topology.

A. Total Channel Consumption

Total channel consumption (TCh) is the overall number of channels needed to support the offered traffic multiplied by the time interval these channels are actually used. In Fig. 3, we show TCh required by Ph-GBR and GBR for \( N=50 \) and \( 1000 \): Ph-GBR always requires fewer channels. TCh tends to decrease for increasing load, because for high loads there is a higher probability to share. Note that the decrease of \( N \) (which corresponds to an increase of the reprovisioning frequency) from 1000 to 50 leads to a remarkable savings in network resources.

B. Resource Overbuild

A figure of merit for comparing backup resource efficiency is resource overbuild, defined as the amount of wavelength channels consumed by backup paths over the amount of wavelength channels utilized by working paths [21]. Resource overbuild (RO) indicates the amount of extra resources needed for providing protection as the percentage of the amount of resources required without protection. Typically, it is desirable to have lower RO because this implies better backup sharing.

Figures 4(a) and (b) compare the RO performance of Ph-GBR and GBR in the case of \( K =1 \) and \( N=50 \) and \( N=1000 \) and 50.
Fig. 4. Resource Overbuild comparison with $K = 1$ and $K = 3$ for $N = 50$ (a) and $N = 1000$ (b). Percentage gain on Resource Overbuild comparison with $K = 1$ and $K = 3$ (c).

and 1000. Ph-GBR always outperforms the respective heuristic GBR when sharing of backup resources is involved.

Fig. 4(c): the savings in RO are remarkable especially for higher values of $K$ and light offered traffic. We obtained savings ranging from 2% to almost 10%. Note that $K$ affects positively the RO percentage difference also because, when more than one path pair is evaluated, the working-path average length tends to increase.

We can observe that PHOTO’s gain tends to decrease for high values of the arrival rate. This behavior is due to the same nature of the algorithm: Ph-GBR tries to give a suggestion on the best route for the backup path on the basis of information on the holding time of the existing connections and neglecting future connection arrivals. Hence, if the arrival rate is high then the quality of the estimation used by the algorithm decreases which, in turn, reduces the value of Ph-GBR over GBR.

C. Ph-GBR in presence of failures

Finally, we have carried on simulative experiments to evaluate Ph-GBR performance in presence of link failures. We have considered that a link fails, on average, each 1000 connection requests and we have evaluated the probability that all connections affected by a link failure are restored (with a working and a backup path). We report the average Reprovisioning Success Ratio of GBR and Ph-GBR:

<table>
<thead>
<tr>
<th>AR</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSR_{GBR}</td>
<td>0.989</td>
<td>0.989</td>
<td>0.986</td>
<td>0.984</td>
<td>0.976</td>
</tr>
<tr>
<td>RSR_{Ph-GBR}</td>
<td>0.990</td>
<td>0.990</td>
<td>0.988</td>
<td>0.986</td>
<td>0.983</td>
</tr>
</tbody>
</table>

Ph-GBR succeeds in maintaining (or even slightly improving) the same RSR of GBR after a link failure, while achieving better backup resource savings. For sake of brevity, we do not present other results regarding the link failure scenario.

VIII. CONCLUSION

In this paper we propose to exploit the knowledge of connection holding time, among the other SLS, to develop a novel intelligent approach for backup reprovisioning (BR). We have provided, for the first time, an ILP formulation for the holding-time-aware BR problem. Our holding-time-aware, dynamic, backup reprovisioning algorithm improves sharing of backup resources while keeping unchanged its ability to overcome multiple failures. We observed significant savings in protection-resource usage by employing our new approach, called Ph-GBR, as opposed to another efficient approach, namely, GBR. The improvement in resource overbuild is found to be up to 10% for a US nationwide network with typical parameters. The proposed method is applicable to other contexts as well, such as MPLS networks, for bandwidth-guaranteed connections.

APPENDIX

To prove NP-completeness of SPP-BR by component design [22], we set $B$ at a given value and then we can formulate the question of decision problem as in Section IV.

Theorem: SPP-BR is NP-complete.

Basic Idea on Our Proof of Theorem: We reduce 3SAT, which is known to be NP-complete, to the SPP-BR problem. The 3SAT problem is formally stated as follows.

3SAT Instance: Collection $F = (D_1, D_2, \ldots, D_M)$ of clauses on a finite set $Q$ of variables such that $|D_i| = 3$ for $1 \leq i \leq M$, where clause $D_i$ is the Boolean “or” of exactly three literals (a literal is either a variable or the Boolean “not” of a variable) and is satisfied by a true assignment if and only if at least one of the three literals is true. $|F| = M$, where $M$ is the number of clauses; $|Q| = N$, where $N$ is the number of variables.

3SAT Question: Is there a truth assignment for $Q$ that satisfies all the clauses in $F$?

Given an arbitrary instance of 3SAT we derive an optical network topology as indicated in Fig. 5. To each variable $v_j$, where $1 \leq j \leq N$, corresponds a connection $l_j$ associated at one existing working path $l^w_j \in L_w$ and a request for a backup path $l^b_j \in L_b$. To each clause $D_i = (v_x, v_y, v_z)$, where $1 \leq i \leq M$, corresponds a connection $l^i$ associated at one existing working path $l^{w_i} \in L_w$ and a request for a backup path $l^{b_i} \in L_b$, as illustrated in Fig. 5. A set of the all $|L_b|$ backup paths, each for existing connection, satisfies the SPP constraints (C.1–C.4, see Section III) and the bound $B = 3N + 2M$ if and only if the given instance of 3SAT is satisfied. The formal proof follows.
Proof of Theorem: SPP-BR ∈ NP since a nondeterministic algorithm can guess a set of \(|L_0|\) backup paths, each for existing connection, and checks in polynomial time if this satisfies the bound \(B\) and the SPP constraints with respect to the corresponding working paths \(L_w\). In the rest, we prove that the problem is also NP-complete.

Given an arbitrary instance of 3SAT \(F = (D_1, D_2, ..., D_M)\) and \(Q = (v_1, v_2, ..., v_N)\), we construct in polynomial time an instance of SPP-BR \(G = (V, E, \lambda), L_w\), where \(\lambda(e) = 1\) for all \(e \in E\). In the following construction, we first define the set of nodes \(V\), then define the set of links \(E\), and finally define the set of existing working paths \(L_w\).

The set of nodes \(V\) can be divided into two groups. The first group is related to the variables in \(Q\) and consists of nodes \(s_j, d_j, a_j, b_j, \pi_j\) and \(\overline{\pi}_j\), where \(1 \leq j \leq N\). \(s_j\) and \(d_j\) represent respectively the source node and the destination node of the connection \(l_j\). The second group is related to the clauses in \(F\) and consists of nodes \(s^F_i\) and \(d^F_i\), where \(1 \leq i \leq M\). \(s^F_i\) and \(d^F_i\) represent respectively the source node and the destination node of the connection \(l^F_i\).

The set of links (unidirectional fibers) \(E\) can be divided into three groups. The first group of links will be used to assign values to the variables in \(Q\); the second group of links will be used to evaluate the Boolean value of the clauses; the third group of links will be used for existing working paths. The three groups of links are as follow.

(i) Group 1: \(<s_j, a_j>, <a_j, b_j>, <b_j, d_j>, <s_j, \pi_j>, <\pi_j, b_j>, <b_j, d_j>, 1 \leq j \leq N>\).

(ii) Group 2: \(<s^F_i, a_j> \text{ and } <b_j, d^F_i>\), if and only if the variable \(\pi_j\) is in the clause \(D_i\). Alternatively, \(<s^F_i, \overline{\pi}_j>\) and \(<\overline{\pi}_j, d^F_i>\), if and only if the variable \(v_j\) is in the clause \(D_i\), \(1 \leq j \leq N\) and \(1 \leq i \leq M\).

(iii) Group 3: \(<s_j, d_j>, 1 \leq j \leq N\). \(<s^F_i, d^F_i>, 1 \leq i \leq M>\).

The set of \(|L| = (M + N)\) existing working paths is defined as follows:

(a) \(N\) working paths of connections \(l_j\) on the associated link \(<s_j, d_j>, 1 \leq j \leq N>\).

(b) \(M\) working paths of connections \(l^F_i\) on the associated link \(<s^F_i, d^F_i>, 1 \leq i \leq M>\).

The construction of our instance of SPP-BR is completed by setting a bound \(B = 3N + 2M\). This construction can be accomplished in polynomial time. Now we prove that \(G\) can support a set of \(|L_0|\) backup paths respecting the SPP constraints and the bound \(B\) if and only if \(F\) is satisfiable.

Let \(v_1 = x_1, v_2 = x_2, ..., v_N = x_N\) be a generic assignment for \(F\), where \(x_j \in \{0, 1\}\) for \(1 \leq j \leq N\). The backup paths can be routed as follows. The backup path \(l^F_i\) is routed from node \(s_j\) to node \(d_j\) via the nodes \(a_j\) and \(b_j\) if and only if \(x_j = 0\), that is \(<s_j, a_j, b_j, d_j>\), for each \(1 \leq j \leq N\). Otherwise \(l^F_i\) is routed via the nodes \(\pi_j\) and \(\overline{\pi}_j\). Conversely, the backup path \(l^F_i\) can be routed from node \(s_i\) to node \(d_i\) via the nodes \(\pi_i\) and \(\overline{\pi}_i\), if and only if the variable \(v_j\) is in the clause \(D_i\).

By this construction the SPP constraints are always respected. \(l^F_i \in L_w\) and \(l^F_i \in L_0\) path of each connection \(x\) in the network are link disjoint (C.1) due to the following considerations: \(l^F_i\) traverses link in Group 1 whereas \(l^F_i\) traverses link in Group 3, \(1 \leq j \leq N\); analogously \(l^F_i\) traverses link in Group 1 and 2 whereas \(l^F_i\) traverses link in Group 3, \(1 \leq i \leq M\).

For the same reasons \(l^F_i \in L_w\) and \(l^F_i \in L_0\) of two different connections, \(x\) and \(y\) respectively, do not share any wavelength between each other (C.3). The backup paths \(l^F_i \in L_w\) and \(l^F_i \in L_0\) associated to two different connections, \(x\) and \(y\) respectively, can always share a wavelength on a common link because the \(l^F_i \in L_w\) and \(l^F_i \in L_w\) in the network are surely link-disjoint by construction of Group 3 (C.4). Two different working paths never share the same SRLG. For the same reason two paths \(l^F_i \in L_w\) and \(l^F_i \in L_w\) are not, do not utilize the same wavelengths (C.2).

Now let \(v_1 = x_1, v_2 = x_2, ..., v_N = x_N\) be an assignment that satisfies \(F\), where \(x_j \in \{0, 1\}\) for \(1 \leq j \leq N\). Without loss of generality, consider the clause \(D_i = v_1 \lor v_2 \lor v_3\). There will be three possible backup paths for the connection \(l^F_i\) from node \(s^F_i\) to \(d^F_i\), that are \(<s^F_i, \overline{\pi}_i, b_1, d^F_i>\), \(<s^F_i, a_2, d^F_i>\) and \(<s^F_i, \pi_3, b_3, d^F_i>\). Since \(v_1 = x_1 = x_2 = x_3 = 0\) \(v_1 = x_1\) is an assignment that satisfies \(F\), either \(x_1 = 1\), \(x_2 = 1\), \(x_3 = 0\). If \(x_1 = 1\), the backup path of connection \(l_1\) traverses the nodes \(\pi_1\) and \(\overline{\pi}_1\); if \(x_1 = 1\) the backup path of connection \(l_2\) traverses the nodes \(\pi_2\) and \(\overline{\pi}_2\); if \(x_3 = 0\) the backup path of connection \(l_2\) traverses the nodes \(a_3\) and \(b_3\).

In this example the connection \(l^F_i\) has the chance to share a backup wavelength on \(<s^F_i, \pi_3, b_3, d^F_i>\) with \(l_1\), and consequently a condition of the clause is set true. If more than one condition in the clause is true, then we can
randomly pick one. To prove this we consider the bound $B = 3N + 2M$. If $F$ is satisfied, for our proof we must ensure that the SPP constraints and also the bound are respected. The bound $B$ is respected when our solution presents, for each $l^F_i$ connection, at least a sharing chance, on a link of the Group 1 (e.g., in Fig. 5 on $<a_1, b_1>$ for clause $D_1$), of a backup wavelength with one of the three $l_j$ connections that represent the variables in the clause $D_i$ (i.e., $|j| (v_j \lor \pi_j \lor b_j$ is in $D_i)$). In general we will obtain a total cost of at least $B = 3N + 2M$ backup wavelengths, but only if $F$ is satisfied, the solution presents exactly two backup wavelengths on the links of Group 2 for each $l^F_i$ connection and exactly three backup resources on the links of Group 1 for at least one $l^F_i$ connection, and at least one of these backup wavelengths is used also by $l^F_i$. If $F$ is not satisfiable, we cannot find a solution with $B \leq 3N + 2M$, because it would be necessary at least one “dedicated” wavelength on a link of the Group 1 for at least one $l^F_i$ connection (and this would imply a false value of the clause $D_1$ and then a false assignment for $F$, e.g. as clause $D_2$ in Fig. 5).

So far we have shown that, if $F$ can be satisfiable, we can also satisfy the backup requests of the connections in the network, respecting the SPP constraints and with a the total backup allocation cost less or equal than $B = 3N + 2M$.

Conversely, let us suppose that in the construction-graph $G (L_b)$, by path $F$, that satisfy the SPP constraints and whose total cost for backup allocation is $B = 3N + 2M$, can exist. In the following we show that the corresponding clause set $F$ is satisfiable. If the set $\{L_b\}$ satisfies the SPP constraints and the bound $B = 3N + 2M$, for all $i$ ($1 \leq i \leq M$) a wavelength can be shared between $l^F_j$ and $l^F_i < b_j$, $b_j$ on a link in the Group 2, either a link $<a_j, b_j >$ or $<b_j, \pi_j >$ for a $j (v_j \lor \pi_j \lor b_j$ is in the clause $D_i$).

Let us now represent the backup paths $l^B_i$, where $1 \leq j \leq N$, by a $N$-bit binary numbers $\Gamma \Gamma_1 \Gamma_2 ... \Gamma_N$. If the backup path of the connection $l_j$ is routed over $<s_j, a_j, b_j, d_j>$, $\Gamma_j = 0$; otherwise $\Gamma_j = 1$ (note that there is a one to one mapping between an $N$-bit binary number and the backup routes $l^B_i$ in the network). Conversely, the backup path of $l^F_i$ has three possible routes and takes the form of $<s^F_i, a_j, b_j, d_j >$, or $<s^F_i, \pi_j, b_j, d_j >$, or $<s^F_i, \pi_j, b_j, d_j >$. If and only if the links $<s^F_i, a_j >$, or $<b_j, d^F_i >$ exist, the backup path can transit by $<a_j, b_j >$ and hence the clause $D_i$ contains the variable $\pi_j$; otherwise if the links $<s^F_i, \pi_j >$ and $<\pi_j, b_j >$ exist, the backup path can transit by $<\pi_j, b_j >$ and the clause $D_i$ contains the variable $v_j$. Therefore, the clause $D_i$ is true under the assignment $v_j = \Gamma_j$ if and only if there is a common backup wavelength between $l^F_i$ and $l^F_i < b_j$, $b_j$ on a link $<a_j, b_j >$ or $<b_j, \pi_j >$. Let us prove this last assumption. Without loss of generality, let $l^F_i$ have the chance to transit via $<a_1, b_1 > \lor <a_2, b_2 > \lor <\pi_3, b_3 >$. Then the clause $D_i$ will be defined by $D_i = \pi_1 \lor \pi_2 \lor \pi_3$. $l^F_i$ has a possibility of sharing if and only if at least one of the backup paths $l^B_i, l^B_i$ traverses $<a_1, b_1 >$ or $<a_2, b_2 >$ or $<\pi_3, b_3 >$ respectively. As a consequence, the clause $D_i$ will be true because at least one among $\Gamma_1, \Gamma_2$ or $\Gamma_3$ will be set to 0, 0 or 1, respectively. Since the backup paths satisfy the SPP constraints and the bound $B = 3N + 2M$, $F$ is true under the following assignment: $v_j = \Gamma_j, 1 \leq j \leq N$, where $\Gamma_1 \Gamma_2 ... \Gamma_N$ is the $N$-bit number corresponding to the backup $l^F_i$.

This concludes our proof that GBR is NP-complete.

REFERENCES