Minimum Cost Optoelectronic Networks: The Optics/Electronics tradeoff

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Abstract—We propose a design model for optoelectronic networks based on DWDM technology. We give mathematical formulations for the logical layer design problem and for the wavelength assignment and routing problem. We study the tradeoff between optical cross-connects and electronic muxes that should be deployed in the various PoPs in order to minimize the cost of the network. We show that cost savings can be significant when we use both electronic muxes and optical cross-connects compared to the all-optical solution. We also show that the savings increase considerably with the traffic volume.

I. INTRODUCTION

We consider the design problem for an optoelectronic network based on Dense Wavelength Division Multiplexing (DWDM) technology. The network is owned by a carrier who wants to satisfy the traffic demands from his customers while minimizing the cost of the transmission equipment he needs to acquire. The demands are typically from small carriers, ISPs and enterprises wishing to interconnect their different sites or expand their own networks by leasing capacity from the carrier.

We consider the logical topology, the network access and the wavelength assignment and routing optimization problems. The logical topology optimization consists of finding the optimal number of channels (wavelengths) that should be allocated between each pair of nodes or PoPs (Points of Presence) [5]. The network access problem consists of finding out whether the traffic demands should be routed directly on dedicated optical channels or should be multiplexed using electronic muxes which is known as the traffic grooming problem [7]. The wavelength assignment and routing problem consists of optimizing the channel routing on the physical network [4].

One of the earliest WDM networks design studies is given in [1] where the authors show that the choice of the physical topology does not have a major effect on the delay or throughput that can be attained through the optimization of logical topology. In [6], [4], the authors show that wavelength converters may not be needed in practice.

A work that is more or less related to ours is proposed in [3]. The authors propose a cost model, that takes only optical components into account however, and study resource budgeting tradeoffs in the allocation of transceivers per node on one hand, and wavelengths per fiber on the other hand.

In this paper, we rather consider the tradeoff between the number of transceivers per node on one hand, and the number of electronic muxes per node on the other hand.

In fact, although more and more efficient optical technology is being introduced in transmission networks, electronics are still dominating the market and therefore still have a major impact on the cost of the deployed technology. Our objective here is to determine the tradeoff between optical components (mainly transceivers) and electronic components (mainly muxes) in order to minimize the investment in the transmission equipment.

In [2] we described some key issues for the optoelectronic flat networks design problem. We assumed that the physical link capacities can take any continuous value. We also proposed a cost model where the cost of electronics evolves as a stair case function with the size of the switches. Using that cost model, we have shown that it is economical to multiplex small traffic demands using electronics and to use direct optical access for large demands. We have also shown that the more degree constraint on the nodes is stringent, the higher the cost of the network. We have also shown that multihopping yields significant savings when the cost of optics is high.

In this paper, we propose a more pragmatic model where the cost of the network increases with the number of optical transceivers and electronic muxes. We assume that the physical link capacities have predefined fixed rates as in SONET.

The paper is organized as follows. In section II, we present the network design and cost models. In section III, we give a mathematical formulation for the logical layer design problem and for the wavelength routing problem. In section IV, we give some numerical results. Conclusions are given in section V.

II. THE MODEL

We assume that the network is composed of optical cross-connects, electronic muxes, fiber links and various optical equipment such as wavelength converters and amplifiers.

A. Network Architecture

We assume that we are given the physical network that includes the Points of Presence (PoPs) and the fiber links connecting them. We assume that the physical network topology is based on a dual-ring.

One of our objectives is to determine the PoPs where we need only optical cross-connects (no demand (de)multiplexing) and the PoPs that require also electronic muxes.
The logical network is composed of the wavelengths routed on top of the physical network. A second objective of ours is to optimize the logical topology i.e. determine the optimal number of channels between each pair of PoPs.

B. Network Access

We assume that the traffic demands are known which is the case in practice for this type of problems. Typically the traffic demands are from small carriers such as ISPs and businesses wishing to buy or lease capacity from big carriers and this type of demand is usually known in advance through the marketing and sales department. We assume that traffic demands can access the network via electronic or optical interfaces.

When a given traffic demand access is via electronics, multiplexing it with other demands (on the same wavelength) is possible. If however the traffic demands access the network via the optical domain, then an entire wavelength is used for one channel per customer. This suggests that we may need electronic multiplexing of the small traffic demands.

A third objective is then to determine the optimal network access i.e. via electronic multiplexing or directly through optics.

C. Cost model

We assume that the cost of the network is given by the sum of the costs of optical transceivers (two per optical channel) and the cost of electronic muxes. We assume that we are able to include the cost of all related equipment such as power supplies, cabling, patch panels, racks etc in the costs of the transceivers and the muxes. Note that the wavelength converters and optical amplifiers cost is taken into the costs of the transceivers and the muxes. Note that such as power supplies, cabling, patch panels, racks etc in we are able to include the cost of all related equipment (channel) and the cost of electronic muxes. We assume that traffic demands can access the network via electronic muxes; and sales department. We assume that traffic demands can introduce the number of optical amplifiers and wavelength converters. When a given traffic demand access is via electronics, multiplexing it with other demands (on the same wavelength) is possible. If however the traffic demands access the network via electronic muxes then an entire wavelength is used for one channel per customer. This suggests that we may need electronic multiplexing of the small traffic demands.

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III. MATHEMATICAL FORMULATION

We will formulate the optoelectronic network design problem as a mixed integer problem. We define:

- $W$: maximum number of wavelengths per fiber (depends on the technology)
- $B$: Capacity of one channel (2.5 Gbit/s)
- $d^{i,j}(t)$: static traffic demand of index $t$ from location $i$ to location $j$ (in bits/sec)
- $n_{k,l}$: number of optical channels between locations $k$ and $l$
- $N_k$: Number of electronic muxes in location $k$
- $\omega_{k,l}(n_{k,l})$: cost of having $n_{k,l}$ optical channels on link $(k,l)$
- $\eta_k(N_k)$: cost of having $N_k$ muxes in location $k$
- $\alpha^{i,j}(t)$: indicator variable: $\alpha^{i,j}(t) = 1$ if demand of index $t$ from $i$ to $j$ access the network via electronic muxes; and $\alpha^{i,j}(t) = 0$ if the access is directly through optics.

To ease the notations, let $\beta^{i,j}(t) = 1 - \alpha^{i,j}(t)$.

- $x^{i,j}_{p}(t)$: fraction of demand $d^{i,j}(t)$ offered to path $p$
- $f^{i,j,p}(k,l)$: path-arc incidence matrix.
- $D_k$: maximum number of channels for node $k$. This is imposed by the cross-connect and/or mux speed.
- $t_k$: number of transmitters per node $k$, $\sum_i x_{k,l} = t_k$
- $r_k$: number of receivers per node $k$. $\sum_t x_{k,l} = r_k$

We have the following relations:

$$n_{k,l} = \left[ \frac{1}{B} \sum_{p, i, j, t} x^{i,j}_{p}(t) f^{i,j,p}(k,l) \right] + \sum_{p, i, j, t} \beta^{i,j}(t) f^{i,j,p}(k,l)$$

and

$$N_k = \left[ \frac{1}{B} \sum_{i, t} \omega_{k,l}(n_{k,l}) + \sum_k \eta_k(N_k) \right] + \left[ \frac{1}{B} \sum_{i, t} \omega_{k,l}(n_{k,l}) + \sum_k \eta_k(N_k) \right]$$

where $L(k)$ is the set of logical multihop paths traversing location $k$.

Using $n$, $x$ and $\alpha$ as independent variables, we can formulate the logical layer design problem as

$$\min_{n, x, \alpha} z = \sum_{k, l} \omega_{k,l}(n_{k,l}) + \sum_k \eta_k(N_k)$$

$$\sum_p x^{i,j}_{p}(t) = d^{i,j}(t) \alpha^{i,j}(t)$$

$$x^{i,j}_{p}(t) \geq 0$$

$$n_{k,l} \in \{0, 1, 2, 3, \ldots\}$$

$$n_{k,l} \geq \frac{1}{B} \sum_{i, j, t} x^{i,j}_{p}(t) f^{i,j,p}(k,l) + \sum \beta^{i,j}(t) f^{i,j,p}(k,l)$$

$$\alpha^{i,j}(t) \in \{0, 1\}$$

$$t_k \leq D_k$$

$$r_k \leq D_k$$

The objective function (1) is given by the sum of the costs of optical channels and electronic muxes. Equation (2) is the conservation constraint. Constraint (5) assures that we have enough channels between locations $k$ and $l$. Constraints (7–8) are the node degree constraints.

A. Channel routing and wavelength assignment

Let $J_{m,n} = 1$ if at least one channel is routed on fiber $(m,n)$, 0 otherwise. The objective is to minimize the number of fibers used i.e. $\sum_{(m,n)} J_{m,n}$. This in turn will minimize the number of optical amplifiers and wavelength converters.

Let $a_{k}$ be the number channels routed on physical path $(k, i, j)$ i.e. path number $k$ between nodes $i$ and $j$. We introduce $p_{m,n}$ as the number of channels using fiber $(m,n).$
The channel routing and wavelength assignment problem is:

\[
\min z = \sum_{(m,n)} \mathcal{J}_{m,n} \quad (9)
\]

\[p_{m,n} = \sum_{(k,i,j)|(m,n)\in(k,i,j)} a_{k}^{i,j} \quad (10)
\]

\[\sum_{k} a_{k}^{i,j} = n_{i,j} \quad (11)
\]

where \(n_{i,j}^{*}\) is the optimal number of channels between nodes \(i\) and \(j\) obtained by solving problem (1–7) and \(W\) is the maximum number of wavelengths per fiber.

B. Problem Resolution

Problem (9–12) is an integer problem that can be solved relatively easily using CPLEX for instance. In fact, usually carriers do not optimize the channel routing and wavelength assignment very frequently typically, on a monthly basis. As such, optimization tools that require a significant amount of computation time can safely be used.

Nevertheless, network planning departments may need faster resolution tools in order to make quick and accurate decisions in a competitive market.

We noticed that the computation time using CPLEX increases rapidly with the number of nodes and with the number of demands. For example, an 8 nodes network with 75 demands required about two hours of computation.

We are currently working on a fast heuristic to solve both the network access and the logical topology design problems and results will be published shortly.

IV. NUMERICAL RESULTS

Consider the dual fiber ring network of four nodes depicted in figure 1. The nodes represent large points of presence (PoPs) where DWDM and electronic muxes could be collocated.

![Fig. 1. Dual-Ring Four-Node Network](image)

The traffic demands are given in table I. We assume that for each demand we can use either the direct optical channel or a multihop path (with a single transit node). One of the objectives here is to find which path to use for each demand.

We assume that \(\omega\) represents the ratio of the cost of an optical transceiver to the cost of an electronic mux.

We solve the problem using CPLEX. The optimal routing, the wavelength assignment and the network access are given in figures 2–6 for different values of \(\omega\) where \(N_{o}\) is the total number of channels and \(N_{e}\) is the total number of electronic multiplexers. We can see that three wavelengths are used when \(\omega \leq 1\) while only two wavelengths are used when \(\omega > 1\).

Note that when \(\omega \to \infty\) the optimal solution is the one given for \(\omega \geq 2\) as it is not possible to further reduce the number of channels.

Consider the optimal solution for \(\omega = 1\) given in figure 4. We can see that nine demands access the network through electronic muxes while six demands use direct optical channels. In addition, we can see the demands 1 \(\to 3\) and 2 \(\to 1\) use the multihop logical paths 1-2-3 and 2-3-1, respectively. This means that it is advantageous to allow the use of multihopping as this gives some flexibility in finding a routing that reduces the cost of the network.

Assume that the carrier opts for an all-optical solution no matter the cost of optics (\(\omega\)). He would then use exclusively optical cross-connects which corresponds to the network given infigure 4.

![Fig. 2. Optimal solution for \(\omega \leq 0.5\). \(N_{o} = 15, N_{e} = 0\): all optical solution, three \(\lambda_{s}\) used.](image)
in figure 2. This means that if the carrier uses that configuration even if $\omega = 2$ for instance, then he would use 15 optical channels i.e. 30 transceivers and no electronic muxes; instead of 16 optical transceivers (i.e. 8 channels) and 14 electronic muxes which correspond to the optimal solution for $\omega \geq 2$ in figure 6.

Recording that when $\omega = 1$ one optical transceiver has the same price as one electronic mux, when $\omega = 2$, one optical transceiver costs twice the cost of an electronic mux. Thus, if the carrier opts rather for the optimal solution given for $\omega \geq 2$ in figure 6, the savings would be 23.3%.

We give in table II the savings obtained using the optimal solution instead of the all-optical solution for different values of $\omega$. Note that when $\omega \to \infty$ the saving is given by $100 \times (1 - N_{o \text{min}} / N_{o \text{max}})$ where $N_{o \text{min}}$ is the optimal number of optical channels obtained with the maximum electronic multiplexing and $N_{o \text{max}}$ is the optimal number of channels obtained when no electronics are used. Note that $N_{o \text{max}}$ is equal to the number of demands since in the all-optical solution, we use one optical channel per demand.

As shown in table II, the savings increase considerably with $\omega$. This is rather expected but indicates that carriers can save considerable amounts of cash by using both electronic muxes and optical transceivers instead of an all-optical solution.

We have also tested the savings for different traffic patterns and network sizes. In order to isolate the effect of the number of customers on one hand and the number of PoPs on the other hand, we chose the values of the demands such that the overall traffic carried by the network is roughly the same.

The evolution of the savings vs the number of demands is given in table III. As we can see in that table, the savings...
significantly increase with the number of demands.

On the other hand, the evolution of the savings vs the number of nodes is given in table IV. As we can see, the savings decrease with the number of nodes especially for small values of $\omega$ and a small number of demands. This is because for a large number of nodes and a small number of demands, the possibility to multiplex demands are limited and as such, the savings obtained by multiplexing are rather small.

When the number of demands is large however, the savings are significant even for large networks. In fact, with 8 PoPs and 50 demands, we have 52% cost savings when we use electronic muxes compared to the case where we have an all-optical network (for $\omega = 2$).

This suggests that instead of considering the number of nodes ($N_\omega$) or the number of demands ($N_d$) separately, we should rather consider the ratio $\rho = N_d/N_\omega$. As we can see in figure 7, the savings increase significantly with $\rho$.

Note that in practice, it is not common to have expensive DWDM and high speed electronic transmission equipment in each PoP. Typically only large PoPs are equipped with such equipment (as we could notice on a number of transmission networks). Therefore, 8 PoPs is a reasonable number for our problem.

V. CONCLUSION

We proposed a network design model for optoelectronic networks based on DWDM technology. We gave a mathematical formulation for the logical layer design problem. We also proposed a mathematical formulation for the wavelength assignment and routing problem.

We studied the tradeoff between optical transceiver and electronic muxes that should be deployed in the various PoPs in order to minimize the cost of the network.

We have shown that cost savings can be significant when we use both electronic muxes and optical cross-connects compared to the all-optical solution. We have also shown that the cost savings increase considerably with the ratio of the number of demands to the number of nodes.

We have explained why carriers should not opt systematically for an all-optical solution as electronics can still save them considerable amounts of cash while responding to their needs in terms of capacity.

The next step for this work is to propose heuristics to solve the network access, logical layer design and wavelength routing problems. We are currently developing such heuristics and results will be published soon.

Another issue is to study the case where the demands are not fixed but rather evolve with time according to some factors such as the economic situation. This yields a dynamic programming problem where we must determine not only the type and the number of components that need to be acquired, but also when these need to be acquired.

REFERENCES


