Abstract—In this paper, a novel algorithm optimizing the utilization of backup path resources for survivable WDM mesh grooming networks, based on graph vertex-coloring approach, is proposed. This is the first optimization technique, dedicated to protection-at-connection level (PAC) in WDM grooming networks, such that does not increase the backup path length and thus provides fast service recovery.

The concept was evaluated for the U.S. Long-Distance Network and European COST 239 Network. The results show that, with only a little degradation of link capacity utilization efficiency (up to 8%), up to 20% shorter average values of service recovery time can be achieved.

I. INTRODUCTION

This paper investigates the issues of service survivability in optical transport networks, where, due to wavelength division multiplexing (WDM), each network link consists of a set of non-overlapping channels (wavelengths), each one capable of transmitting the data independently at speed of a few Gbps. Any network element failure may thus lead to large data and revenue losses [1]. In WDM networks a demand to perform the data transmission between a given pair of nodes is served by establishing an end-to-end connection, referred to as a lightpath [2]. Each lightpath is realized by allocating a sequence of wavelengths at consecutive links. In case of imposed wavelength continuity constraint, the lightpath must utilize the same wavelength on each link it traverses. Otherwise, wavelength conversions may be performed at lightpath transit nodes.

Network survivability, originally defined as the capability of a system to fulfill its mission in a timely manner, in the presence of attacks, failures or accidents [3], is achieved by introducing redundancy. It means that for the main path of a WDM connection, referred to as active path, there are additional paths, called backup paths, used to protect the connection in case of a certain failure scenario [2, 4, 5]. Single network element failures are typically considered. Survivability is based on either dedicating backup resources in advance (protection scheme) or on dynamic restoration [2, 5]. Regarding the scope of a backup path, path, region or link protection/restoration is typically used [2, 4].

There is a significant difference between the typical bandwidth requirement of the end-user traffic demand (e.g. several Mbps) and the capacity of a WDM wavelength (e.g. 10 Gbps). Since each WDM connection normally occupies the whole wavelength of a link, in order to avoid a large waste of link capacities, several low speed traffic streams need to be multiplexed together (i.e. groomed) onto a given wavelength, using time division multiplexing (TDM) [6]. These streams are then carried jointly by a given lightpath, however at different subchannels.

In WDM grooming networks, survivability of connections is provided at either connection or lightpath level. In protection-at-connection level (PAC) [6] operating at a per-flow basis, each active path is protected by a single end-to-end backup, as shown in Fig. 1. On the contrary, protection-at-lightpath level (PAL) approach [6] allows each connection to be realized by a sequence of active lightpaths, each active lightpath protected by a dedicated backup lightpath.

Fig. 1. An example of protection-at-connection level (PAC). The active paths of connections (3, 9) and (3, 13) are realized by active paths $a_1$ and $a_2$, respectively. The backup paths $b_1$ and $b_2$ provide end-to-end protection with regard to the whole active paths $a_1$ and $a_2$, respectively. Active paths $a_1$ and $a_2$ may be groomed onto a common wavelength at links (3, 4) and (4, 5) as well as backup paths $b_1$ and $b_2$ may be groomed at links (3, 6) and (6, 9).

Assuring survivability of connections by establishing backup paths increases the ratio of link capacity utilization, which in turn limits the number of connections that may be established. This ratio can be reduced by applying sharing the link capacities that are reserved for backup paths [2, 7]. Such sharing is possible, if these backup paths are to protect against different failure scenarios (i.e. if the respective protected parts of active paths are mutually disjoint1).

However, typical a priori optimization approach, described in [7], results in increasing the backup path lengths, compared to the “no optimization” case. Regardless of the real link length, each link cost is set here to 0, if there is 1 Depending on the kind of protection (either against a single link or a single node failure), these parts of active paths must be mutually link- or node-disjoint, respectively, meaning that the paths have no common links (transit nodes), respectively.
Typical a priori optimization is performed before finding backup paths, when calculating the cost of $\xi_{ij}$ of each physical WDM link $l_{ij}$ to be used in a backup path $k$, according to Eq. 1 [7].

$$
\xi_{ij} = \begin{cases} 
0 & \text{if } r^{(k)} < m^{(k)} \\
(r^{(k)} - m^{(k)}) \cdot s_{ij} & \text{if } r^{(k)} \geq m^{(k)} \text{ and } f_{ji} \geq r^{(k)} - m^{(k)} \\
\infty & \text{otherwise}
\end{cases}
$$

where:

- $r^{(k)}$ - the requested capacity,
- $m^{(k)}$ - the capacity reserved so far at a link $l_{ij}$ (for the backups of the already established active paths) that may be shared,
- $f_{ji}$ - the unused capacity of a link $l_{ij}$
- $s_{ij}$ - the length of a link $l_{ij}$

However, the found backup paths are not the shortest ones here, since the calculated costs of links are often not equal to the link lengths. Any link $l_{ij}$ even a long one, may be used in a backup path, since its cost $\xi_{ij}$ is set to 0, if only its capacities may be shared. Long backups make in turn the process of connection restoration time-consuming.

The proposed optimization of resource utilization, given by the FSR-SCL-VC algorithm, is performed after establishing the connections. Its scope is confined to each single link $l_{ij}$, so sharing the backup paths is performed within the set of backup paths installed at wavelengths of a given link $l_{ij}$ only. This in turn implies that the backup paths, originally established as the shortest ones, remain unchanged, i.e. they use the same links which they used before applying the optimization. Since the length of the backup paths is not increased, fast service restoration is possible. The proposed FSR-SCL-VC algorithm is executed at each network link $l_{ij}$ independently. For each link $l_{ij}$ it tries to reorganize the initial assignment of subchannels to backup paths by dividing the set of backup paths $B_{ij}$ into subsets $B_{ij}^x$. Each subset $B_{ij}^x$ is to contain backup paths that protect mutually disjoint parts of active paths and thus may share a common subchannel. The number of subsets $B_{ij}^x$ must be minimized, since it denotes the number of subchannels that will become allocated at $l_{ij}$ for backup paths after applying the optimization. The problem of optimally dividing the set $B_{ij}$ of backup paths installed at each link $l_{ij}$ into subsets $B_{ij}^x$ (Step 1 of Algorithm FSR-SCL-VC) is NP-complete as it equivalent to the vertex-coloring problem of an induced graph of conflicts $G_{ij}$, which is also NP-complete [10].

Generally, in such a graph of conflicts $G = (V, E)$ there is an edge between any two vertices $v$ and $w$, if and only if there is a conflict between them. The objective is to find a partition of vertices of $G$ into a minimal number of subsets of mutually compatible vertices, i.e. into subsets of pair-wise non adjacent vertices. As defined in [10], a (proper) coloring of vertices of a graph $G$, is a mapping $f: V \rightarrow C$, where $V$ is a set of vertices of $G$ and $C$ is a finite set of colors, each color being represented by an integer number such that neighboring vertices are assigned different colors (i.e. if $\forall v \in G$ then $f(v) \neq f(w)$).
Input
Network topology; the sets \( B_k \) of backups installed on subchannels of links \( l_k \); the sets of free subchannels at each link \( l_k \); the sets of subchannels allocated for active paths at links \( l_k \).

Output
The sets \( B_k^{'} \) that determine the new assignment of subchannels to backup paths at each link \( l_k \).

For each link \( l_k \):

Step 1. Divide the set \( B_k \) of backup paths, installed at subchannels of a link \( l_k \), into subsets \( B_k^{'} \) such that:
- each subset contains backup paths that may share a given subchannel one another,
- the number of subsets \( B_k^{'} \) is minimized.

Step 2. For each subset \( B_k^{'} \):
Step 2a. delete original subchannel allocations for the backups of \( B_k^{'} \),
Step 2b. apply sharing by allocating one common subchannel for all the backups of \( B_k^{'} \).

*apart from including the constraint on mutual disjointedness of the protected parts of active paths, include the respective constraints on the chosen optimization strength (i.e. for intra-demand, inter-demand or parallel intra- and inter-demand sharing).

Fig. 2. FSR-SCL-VC algorithm

In FSR-SCL-VC algorithm, colors \( c \) are replaced by pairs of colors \((c, d)\), where \( c \) is a wavelength number and a subcolor \( d \) is the \( d \)th subchannel of a wavelength \( c \). For each link \( l_k \) an induced graph \( G_{ij} \) is constructed such that:
- its vertices denote backup paths that are installed at channels of a link \( l_k \),
- there is an edge between a given pair of vertices \( u \) and \( v \) in \( G_{ij} \), if and only if there is a conflict between the respective backup paths (i.e. when the protected parts of active paths are not mutually disjoint), implying that these backups must be assigned different units of bandwidth \((c, d)\).

It turns out that in case of full wavelength conversion capability, no strict assignment of subchannels to certain backups is needed at each link \( l_k \) and that under PAC it is sufficient to calculate the minimum number of subchannels required under backup path capacity sharing. However, this problem is still NP-complete, since it is equivalent to the problem of finding the chromatic number \( \chi(G_{ij}) \) [10] being here the smallest number of pairs of colors \((c, d)\) required to color the vertices of \( G_{ij} \). One of the bounds on \( \chi(G_{ij}) \) are:

\[
\omega(G_{ij}) \leq \chi(G_{ij}) \leq \Delta(G_{ij})+1
\]

where \( \Delta(G_{ij}) \) is the maximum degree of \( v \) in \( G_{ij} \) (i.e. the number of edges incident to \( v \)) and \( \omega(G_{ij}) \) is the maximum size of the fully connected subgraph of \( G_{ij} \), i.e. clique [10]).

However, using the upper bound of \( \Delta(G_{ij})+1 \) to estimate the number of subchannels needed at \( l_k \) may give the results far greater from the optimal ones. For instance, if \( G_{ij} \) has a topology of a star [10] with \( n = 9 \) vertices, meaning that \( \Delta(G_{ij}) = n-1 \) and \( \omega(G_{ij}) = 2 \) then, when using the upper bound of \( \Delta(G_{ij})+1 \), \( n = 9 \) subchannels will be used instead of the sufficient two subchannels. That is why even under full wavelength conversion capability, knowledge of \( \chi(G_{ij}) \) is necessary. What is more, in order to obtain the value of \( \chi(G_{ij}) \), one must always color the vertices of a \( G_{ij} \), meaning that a graph vertex-coloring of \( G_{ij} \) is also returned.

In Sections II.A and II.B the algorithms of both finding the optimal as well as the approximate values of \( \chi(G_{ij}) \), respectively, to determine the minimal number of subchannels needed for backup paths at each link \( l_k \) (Step 1 of FSR-SCL-VC algorithm) for protection at connection level and full wavelength conversion capability, are introduced. They are executed for each link independently and utilize the graph vertex-coloring routine. Apart from returning the value of \( \chi(G_{ij}) \), they also find the strict subchannel assignment with respect to backup paths of \( l_k \).

A. ILP Model of Sharing the Backup Capacities for Traffic Grooming (FSR-SCL-VCO)

Indices
\( k \in \{1,2,\ldots,K\} \) numbers of backup paths in a network, \( \{c,d\} \) pairs of colors to be assigned; \( c = 1,2,\ldots, C \), \( d = 1,2,\ldots, D \); For a given \( D \), set:

\[
C = \left\lfloor \frac{\Delta(G_{ij})+1}{D} \right\rfloor
\]

Variables
\( x_k^{c,d} \) takes value of 1, if a \( k \)th backup path is assigned a pair of colors \((c,d)\) at a given link \( l_k \), 0 otherwise
\( b_k^{c,d} \) takes value of 1, if a pair of colors \((c,d)\) is assigned to any vertex of \( G_{ij} \), 0 otherwise

Objective
For a given graph of conflicts \( G_{ij} \) of a link \( l_k \) it is to find optimal backup path bandwidth sharing while minimizing the total number \( F \) of used subchannels:

\[
F = \sum_{c=1}^{C} \sum_{d=1}^{D} b_k^{c,d}
\]

Constraints
- on assigning each vertex \( k \) of a given graph \( G_{ij} \) one pair of colors \((c,d)\) only

\[
\sum_{c=1}^{C} \sum_{d=1}^{D} x_k^{c,d} = 1; \quad k = 1,2,\ldots,K
\]

- assuring that neighboring vertices of a graph \( G_{ij} \) receive different colors

\[
x_k^{c,d} + x_m^{c,d} \leq b_k^{c,d}
\]

for each edge \( e = (k,m) \) in \( G_{ij} \); \( c = 1,2,\ldots, C \); \( d = 1,2,\ldots, D \);

- excluding the active path subchannels

\[
\sum_{k=1}^{K} \sum_{c=1}^{C} \sum_{d=1}^{D} x_k^{c,d} = 0
\]

if \( x_k^{c,d} \) denotes a subchannel \( d \) of a channel \( c \) allocated at link \( l_k \) for an active path \( k \)

- on taking the allowed values

\[
x_k^{c,d} \in \{0,1\}; \quad b_k^{c,d} \in \{0,1\}
\]

\( k = 1,2,\ldots,K; \quad c = 1,2,\ldots, C; \quad d = 1,2,\ldots, D \);
B. Heuristic Algorithm of Sharing the Backup Capacities for Traffic Grooming (FSR-SCL-VCH)

Since the problem formulated in Section II.A is NP-complete, the following FSR-SCL-VCH heuristic algorithm is proposed here.

**INPUT**
Network topology; the sets \( B_i \) of backup paths installed on subchannels of links \( l_i \); the sets of free subchannels at links \( l_i \); the sets of subchannels originally allocated for active paths at links \( l_i \).

**OUTPUT**
The sets \( B_i^* \) that determine the new assignment of subchannels to backup paths at links \( l_i \).

For each link \( l_i \):

**Step 1.** Create graph of conflicts \( G_{ij}^* \).

**Step 2.** Divide the set \( B_i \) of backup paths, installed on channels of a link \( l_i \) into subsets \( B_i^* \) by assigning the vertices pairs of colors \((c, d)\) using the LF algorithm.

**Step 3.** Delete original subchannel allocations for the backup paths of \( l_i \) and allocate the respective number of subchannels (calculated in Step 2) for backup paths.

The sets \( B_i^* \) are sufficient to provide for all the connections. The first value of \( d \), implying that only one assigned pair \((c, d)\) had the same degree values and then sequentially assigned the lowest possible pair of colors \((c, d)\) based on their position in this ordering, meaning that vertices of higher degree receive their colors first.

Suppose that at a link \( l_i \), there are five backups (using five subchannels) with conflicts as given in Fig. 4. After executing the LF algorithm, two pairs of colors \((c, d)\) were sufficient to color the vertices of \( G_{ij} \). Each assigned pair \((c, d)\) had the same value of \( c \), implying that only one wavelength will be finally needed for backup paths. The first subchannel, such that is not allocated for any active path at \( l_i \), will be reserved for backup paths of connections: 1 and 5, while backup paths of connections 2, 3, and 4 will utilize the common next subchannel such that originally not used for active path purposes.

**TABLE I**

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>EXAMPLE VERTEX-COLORING OF ( G_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>descending ordering of ( G_{ij} ) vertices</td>
<td>1 3 5 2 4</td>
</tr>
<tr>
<td>degrees of ( G_{ij} ) vertices</td>
<td>3 2 1 1 1</td>
</tr>
<tr>
<td>assigned pairs of colors</td>
<td>(1, 1) (1, 2) (1, 1) (1, 2) (1, 2)</td>
</tr>
</tbody>
</table>

**Fig. 3. FSR-SCL-VCH algorithm**

As for the graph vertex-coloring heuristic algorithm, to be used in Step 2 of FSR-SCL-VCH algorithm, Largest First (LF) algorithm, described in [10] is used, since it provides good results in terms of minimizing the total number of utilized pairs of colors \((c, d)\) for the price of acceptably low polynomial computational complexity \(O(n + m))\). In LF, vertices of each \( G_{ij} \) are first ordered descending their degree values and then sequentially assigned the lowest possible pair of colors \((c, d)\) based on their position in this ordering, meaning that vertices of higher degree receive their colors first.

**III. MODELING ASSUMPTIONS**

The experiment was performed for the U.S. Long-Distance Network [11] and European COST 239 Network [12], presented in Figs. 5-6, respectively. Due to NP-completeness of the original problem, only computations using the heuristic FSR-SCL-VCH algorithm were performed. Simulations were to measure the link total capacity utilization ratio, the length of backup paths and the values of connection restoration time. Time of connection restoration was measured according to [5] and comprised: time to detect a failure, link propagation delay, time to configure backup path transit nodes and message processing delay at network nodes (including queuing delay). All WDM links \( l_{ij} \) had \( c = 8 \) channels with grooming capability \( d = 4 \) (each one offering 4 subchannels of equal capacity). Channel capacity unit was considered to be equal for all the network links. Network nodes were assumed to have a full channel (wavelength) conversion capability. For each connection, the following properties were assumed:

- protection against a single link failure,
- protection at connection level (PAC) implying the use of a single backup path for each separate active path,
- LF graph vertex-coloring algorithm for backup path capacity sharing at each link \( l_{ij} \),
- a demand of resource allocation equal to the capacity of one subchannel (equal to the value of one channel capacity divided by \( d \)),
- provisioning 100% of the requested bandwidth after a failure of a network element,
- a demand to assure unsplittable active and backup flows
- the distance metrics and Dijkstra’s shortest path algorithm [9] in all path computations
- the three-way handshake protocol of restoring the connections (the exchange of LINKFAIL, SETUP and CONFIRM messages), described in [5].

The features of the proposed optimization technique were tested using two scenarios of network load. In the first case, the set of connection demands consisted of 10% of all the possible network node pairs, which amounted to 38 and 17 demands for U.S. Long-Distance Network and European COST 239 Network, respectively. Using this scenario there were always enough resources to establish all the demanded connections. The second scenario was intended to test the properties of the proposed optimization technique under varying network load. During a single modeling, shown in Fig. 7, one particular variant of optimization strength was provided for all the connections.
Repeat \( r \) times the following steps:

Step 1. Randomly choose a given number of pairs of source \( s \) and destination \( d \) nodes.

Step 2. Try to establish the survivable connections with protection at connection level using the respective optimization of backup path capacity utilization

Step 3. Store the ratio of link capacity utilization and the lengths of the backup paths

Step 4. \( t \) times simulate random failures of single links. For each failure state restore connections that were broken and note the values of connection restoration time

\( * \) during each investigation, \( r = 30 \) and \( t = 30 \) were assumed

** any type of sharing is allowed here

---

**IV. MODELING RESULTS**

A. Link Capacity Utilization

Figs. 8-9 show the average values of link capacity utilization ratio, measured as the average link total capacity allocated for both primary and protection paths, for all variants of optimization strength ordered ascending the optimization strength. The results prove that the proposed FSR-SCL-VCH algorithm remarkably reduces the link capacity utilization ratio (up to 20%, compared to the “no optimization case”). Although the a priori optimization gives slightly better link capacity utilization ratio in all cases, the difference is practically meaningless.

B. Length of Backup Path

Figs. 10-11 show the average lengths of backup paths for all variants of optimization strength. The results prove that for our FSR-SCL-VCH optimization, the average length of backup paths is not increased and remains at the same level as in case no optimization is performed. This is true independent of the optimization strength. Fast restoration is thus possible. They also show that for the common a priori technique, the backup path length is often far from optimal (up to 33% longer).

C. Values of Service Recovery Time

Figs. 12-13 show the average values of connection restoration time for all variants of optimization strength. Due to not increasing the length of backup paths, the obtained mean values of restoration time for the proposed algorithm were always similar to the shortest ones, achieved when no optimization was performed. When compared to the a priori optimization results, the difference is remarkable. Under PAC, it took up to 20% less time on average to restore a connection, when the FSR-SCL-VCH optimization was used.

---

Fig. 6. European COST 239 Network (19 nodes, 37 links)

Fig. 7. Research plan

Fig. 8. Average ratio of link capacity utilization as a function of optimization strength for U.S. Long-Distance Network

Fig. 9. Average ratio of link capacity utilization as a function of optimization strength for European COST 239 Network

Fig. 10. Average length of backup path as a function of optimization strength for U.S. Long-Distance Network

Fig. 11. Average length of backup path as a function of optimization strength for European COST 239 Network

Fig. 12. Average values of service restoration time as a function of optimization strength for U.S. Long-Distance Network
D. Modeling Results for Varying Network Load

The last experiment was performed for the U.S. Long-Distance Network only. Regarding the strength of optimization, parallel intra- and inter-demand sharing was used. All other modeling assumptions were the same as given in Section III. The modeling was to test the properties of the proposed FSR-SCL-VCH optimization under different network loads. For that purpose, 10 different sizes of demand sets, given in horizontal axes of Figs. 14-16, were used, changing from 10 to 100% with the step of 10%. They corresponded to the average link total capacity consumption changing from 15 to 75%. For instance, the demand set size of 30% meant that each time the connections between randomly chosen 30% of all possible network node pairs were tried to be established.

The results prove that the properties of the proposed FSR-SCL-VCH optimization scale well with the increase of the network load. Independent of the network load, the average ratio of link capacity utilization per connection was only about 8% worse (Fig. 14), but backup path length was always even about 24% shorter (Fig. 15) and finally the connection restoration time values were about 20% shorter (Fig. 16) for FSR-SCL-VCH algorithm, compared to the results of the typical a priori routine. The average link total capacity utilization per connection decreased for both optimization approaches with the increase of the network load (Fig. 14), since the greater the network load is, the more backup paths are likely to share a common subchannel.

V. CONCLUSION

Obtained results confirm that one cannot simultaneously minimize both the values of connection restoration time and the ratio of link capacity utilization. The proposed FSR-SCL-VC algorithm, dedicated to protection at connection level for survivable WDM grooming networks with full wavelength conversion capability, significantly reduces the ratio of link capacity utilization. However, unlike the typical a priori optimization approach, it does not increase the lengths of backup paths and thus also provides fast service recovery. FSR-SCL-VC achieves about 20% shorter average values of connection restoration time at only 8% worse average link total capacity utilization. Future work is to adjust the approach to the case of partial wavelength conversion capability as well as dynamic traffic pattern.

REFERENCES